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Photon Distribution Amplitudes in QCD

PATRICIA BALL¹, V.M. BRAUN², and N. KIVEL²¹*IPPP, University of Durham, Durham DH1 3LE, UK*²*Institut für Theoretische Physik, Universität Regensburg,
D-93040 Regensburg, Germany*

Abstract:

We develop a consistent technique for the calculation of real photon emission in hard exclusive processes, which is based on the background field formalism and allows a convenient separation of hard electromagnetic and soft hadronic components of the photon. The latter ones are related to matrix-elements of light-cone operators in the electromagnetic background field and can be parametrized in terms of photon distribution amplitudes. We construct a complete set of photon distribution amplitudes up to and including twist-4, for both chirality-conserving and chirality-violating operators. The distribution amplitudes involve several nonperturbative parameters and, most importantly, the magnetic susceptibility of the quark condensate. We review and update previous estimates of the susceptibility and also give new estimates of parameters describing higher-twist amplitudes from QCD sum rules.

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1 Introduction

Hard exclusive processes involving photon emission are attracting increasing attention. Examples include transition form factors like $\gamma^* \rightarrow \pi\gamma$ with one real and one virtual photon [1], deeply-virtual Compton scattering (DVCS) [2] and rare radiative B -decays like $B \rightarrow \ell\bar{\nu}_\ell\gamma$, $B \rightarrow \rho\gamma$ and $B \rightarrow K^*\gamma$ [3]. A specific feature of the QCD description of such processes is that a real photon contains both a “pointlike”, electromagnetic (EM), and a “soft”, hadronic, component. This distinction is familiar from the studies of the photon structure functions in deep inelastic scattering (see e.g. [4]), but so far has not been studied, in a systematic way, for exclusive processes. It is the objective of this work to develop an efficient formalism for describing either component of the photon and to give an update of the available information on the hadronic one.

We will work in the background (BG) field formalism. This technique was pioneered by Schwinger for QED [5] and later rediscovered as a convenient tool in QCD, in particular in connection with calculations of the QCD β -function [6], power-corrections to deep-inelastic scattering [7] and QCD sum rules [8, 9]. An excellent technical review can be found in [10]. The principal advantage of the BG field approach is its explicit gauge-invariance which allows the use of different gauges for the soft (classical) and hard (quantum) fields and also avoids contact terms; instead of the latter, the formalism involves new operators containing the EM instead of the gluonic field-strength tensor. It also allows an intuitive physics interpretation which is important by itself. As an example [9], consider quarks and antiquarks in the QCD vacuum, in a constant (electro)magnetic field. In the weak-field limit, the induced magnetisation of the vacuum is proportional to the applied field, the quark density, the quark electric charge e_q and the parameter χ , the so-called magnetic susceptibility of the quark condensate:

$$\langle 0|\bar{q}\sigma_{\alpha\beta}q|0\rangle_F = e_q \chi \langle \bar{q}q \rangle F_{\alpha\beta}. \quad (1.1)$$

Here $\langle \bar{q}q \rangle$ is the quark condensate, $F_{\alpha\beta}$ the field-strength tensor of the external EM field, and the index F indicates that the VEV is taken in the vacuum in the presence of the field $F_{\alpha\beta}$. If the magnetic field is allowed to vary with a certain frequency, the response becomes sensitive to the quark-antiquark separation and gets more complicated. In the limit of light-like separations $z^2 = 0$, which is relevant for hard exclusive processes, the magnetic susceptibility has to be substituted by a response function ϕ_γ [11]:

$$\langle 0|\bar{q}(z)\sigma_{\alpha\beta}q(-z)|0\rangle_F = e_q \chi \langle \bar{q}q \rangle \int_0^1 du F_{\alpha\beta}((1-2u)z)\phi_\gamma(u), \quad (1.2)$$

where the normalization is chosen such that $\int_0^1 du \phi_\gamma(u) = 1$. For a plane-wave configuration $F_{\alpha\beta}(z) \sim \exp(-iqz)$, $q^2 = 0$, and in the infinite-momentum frame $q_+ \rightarrow \infty$, the matrix-element on the l.h.s. of (1.2) describes the probability amplitude that a real photon with momentum q dissociates into a quark-antiquark pair at small transverse separation. The function $\phi_\gamma(u)$ can thus be identified with a photon distribution amplitude (DA) with u and $1-u$ being the momentum fractions carried by the quark and the antiquark in the

photon, respectively, in full analogy with DAs of mesons [12]. To the best of our knowledge, photon DAs have been introduced and studied for the first time in Refs. [11]. Some more results on higher-twist distributions can be found in [13], while in [14] the leading-twist amplitude has been calculated using a chiral quark model in the instanton-vacuum.

The subject of the present paper is the detailed study of these DAs, up to and including twist-4. Most of the discussion will refer to asymptotic real photon states, which correspond to a plane-wave configuration of the EM BG field, whose frequency is, however, not assumed to be small. This deviates from the usual procedure of expanding in slowly varying (classical) fields and requires special techniques which have been worked out in Ref. [15].

The paper is organized as follows: Section 2 is mainly introductory and summarizes notations and basic features of the BG field method. Section 3 contains a detailed discussion of the leading-twist photon DA. Section 4 is devoted to photon DAs of twist-3 and 4. Finally, in Sec. 5 we summarize. The paper also contains several appendices with results of more technical nature.

2 The Background Field Method

2.1 Gauge-invariance and contact terms

The basic idea of the BG field method is to modify the quark part of the QCD action by including an EM field B_μ in addition to the colour gluon field A_μ^a . In order to preserve EM gauge-invariance, the covariant derivative becomes

$$D_\mu = \partial_\mu - igA_\mu^a t^a - ie_q B_\mu, \quad (2.1)$$

where $e_q = eQ_q = \sqrt{4\pi\alpha} Q_q$ with $Q_{u,c,t} = +2/3$, $Q_{d,s,b} = -1/3$ is the quark electric charge and $\alpha = 1/137 \dots$ is the fine-structure constant. The action reads (for one flavour)

$$\mathbf{S}_q = \int d^4x \bar{q}(x)[i\not{D} - m_q]q(x) = \mathbf{S}_q^{\text{QCD}} + e_q \int d^4x B_\mu(x) j^\mu(x), \quad (2.2)$$

$j_\mu = \bar{q}\gamma_\mu q$ is the vector current. The EM field B_μ is treated as purely classical, i.e. it does not participate in loops, and is weak, i.e. we only consider expressions linear in B_μ . In Lorentz-gauge, an *ingoing* photon with momentum q corresponds to the plane-wave field-configuration

$$B_\mu(x) = e_\mu^{(\lambda)} e^{-iqx}. \quad (2.3)$$

Here $e_\mu^{(\lambda)}$ is the photon polarization vector with $e_\mu^{(\lambda)} q^\mu = 0$. The corresponding gauge-invariant EM field-strength tensor is

$$F_{\mu\nu}(x) = i(e_\mu^{(\lambda)} q_\nu - q_\mu e_\nu^{(\lambda)}) e^{-iqx}. \quad (2.4)$$

To avoid confusion, we will always write $F_{\mu\nu}$ for the EM and $G_{\mu\nu} = G_{\mu\nu}^a t^a$ for the gluon field-strength tensors, respectively.

(Almost) any calculation in QCD relies on the separation of scales: the contributions of quark and gluon exchanges with large momenta are calculated perturbatively and

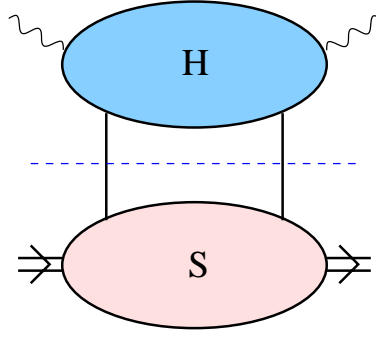


Figure 1: Schematic representation of a hard-soft separation.

form a hard contribution (coefficient function), the contributions with small momenta are parametrized in terms of various distribution functions, alias operator matrix-elements. Such a structure is indicated schematically in Fig. 1. A real photon (not shown) can be emitted from both the hard and the soft block. The advantage of the BG field method relies on the fact that it separates hard and soft photon-emission in an explicitly gauge-invariant way. This is not necessary, but convenient, as we will see below.

Technically, the gauge-invariance is seen most directly if, in the calculation of the hard block, one uses the following expression for the quark propagator in the BG field¹ [15]:

$$S(x) = \frac{i \not{x}}{2\pi^2 x^4} [x, 0]_F - \frac{i}{16\pi^2 x^2} \int_0^1 du [x, ux]_F \{ \bar{u} \not{x} \sigma_{\alpha\beta} + u \sigma_{\alpha\beta} \not{x} \} e_q F^{\alpha\beta}(ux) [ux, 0]_F \\ + \text{terms containing } G_{\alpha\beta}, \quad (2.5)$$

where $P_\mu = iD_\mu$ is the momentum operator, $\bar{u} = 1 - u$ and

$$[x, y]_F = \text{Pexp} \left\{ i \int_0^1 dt (x - y)_\mu [g A^\mu(tx + \bar{t}y) + e_q B^\mu(tx + \bar{t}y)] \right\} \quad (2.6)$$

is the path-ordered gauge-link. Here and below the subscript F serves to indicate that the BG field is included, which means in particular that both the abelian EM and the non-abelian colour-phases are present. We will also use the notation $[x, y]$ without a subscript if the EM field is not included.

The terms in $F_{\alpha\beta}$ generate $U(1)$ gauge-invariant contributions describing photon-emission from small distances, whereas the gauge-factors $[\cdot, \cdot]_F$ in the first term on the r.-h.s. of (2.5) eventually combine with quark fields that connect the hard and the soft block, producing nonlocal gauge-invariant operators. In the simplest situation with no external hadrons, one is left with the vacuum expectation values of such operators in the EM BG field, generically

$$\langle 0 | \bar{q}(x) \Gamma[x, y]_F q(y) | 0 \rangle_F = i e_q \int_0^1 d^4 z B^\mu(z) \langle 0 | T \{ \bar{q}(x) \Gamma[x, y] q(y) j^\mu(z) \} | 0 \rangle$$

¹For simplicity we present the result for the massless quark propagator. For massive quarks, see [16].

$$+ ie_q \int_0^1 dt (x-y)_\mu B^\mu(tx + \bar{t}y) \langle 0 | \bar{q}(x) \Gamma[x, y] q(y) | 0 \rangle. \quad (2.7)$$

Here Γ is an arbitrary Dirac-structure. The first term on the r.-h.s. of (2.7) comes from the expansion of the action e^{iS_q} , cf. (2.2), whereas the second term originates from the gauge-link (2.6). The expression (2.7) is invariant under $U(1)$ gauge-transformations $B_\mu \rightarrow B_\mu + \partial_\mu \phi$. As an example, let us check this for plane-waves. Using (2.3), the r.-h.s. becomes

$$\begin{aligned} & ie_q \int d^4z e_\mu^{(\lambda)} e^{-iqz} \langle 0 | T \{ \bar{q}(x) \Gamma[x, y] q(y) j^\mu(z) \} | 0 \rangle \\ & + e_q e_\mu^{(\lambda)} \frac{(x-y)_\mu}{q(x-y)} [e^{-iqy} - e^{-iqx}] \langle 0 | \bar{q}(x) \Gamma[x, y] q(y) | 0 \rangle. \end{aligned} \quad (2.8)$$

Note that the expression in the first line is nothing but the amplitude of the photon-to-vacuum transition

$$\langle 0 | \bar{q}(x) \Gamma[x, y] q(y) | \gamma^{(\lambda)}(q) \rangle = ie_q \int d^4z e_\mu^{(\lambda)} e^{-iqz} \langle 0 | T \{ \bar{q}(x) \Gamma[x, y] q(y) j^\mu(z) \} | 0 \rangle. \quad (2.9)$$

To check gauge-invariance, replace $e_\mu^{(\lambda)} \rightarrow q_\mu$. The first line in (2.8) then becomes

$$e_q \int d^4z e^{-iqz} \langle 0 | \partial_\mu T \{ \bar{q}(x) \Gamma[x, y] q(y) j^\mu(z) \} | 0 \rangle, \quad (2.10)$$

which is generally nonzero because of contact terms. To calculate the latter, use that inside the QCD functional integral the divergence of the vector current can be replaced by a functional derivative over the quark (antiquark) field:

$$\partial_\mu j^\mu(z) e^{iS_q} = -\bar{q}(z) \left(\frac{\delta}{\delta \bar{q}(z)} e^{iS_q} \right) - \left(\frac{\delta}{\delta q(z)} e^{iS_q} \right) q(z). \quad (2.11)$$

Integrating by parts over the quark fields in the functional integral, one obtains

$$\begin{aligned} & e_q \int d^4z e^{-iqz} \{ \delta^{(4)}(x-z) \langle 0 | \bar{q}(z) \Gamma[x, y] q(y) | 0 \rangle - \delta^{(4)}(y-z) \langle 0 | \bar{q}(x) \Gamma[x, y] q(z) | 0 \rangle \} \\ & = e_q [e^{-iqx} - e^{-iqy}] \langle 0 | \bar{q}(x) \Gamma[x, y] q(y) | 0 \rangle. \end{aligned} \quad (2.12)$$

This is exactly what is obtained from the second term in (2.8), but with the opposite sign. One sees that restoration of $U(1)$ gauge-invariance is achieved through elimination of the contact terms.

Calculation of the hard scattering amplitudes (hard block in Fig. 1) can usually be done for on-shell partons. In the operator product expansion language, this means that operators which vanish by virtue of the QCD equations of motion (EOM) can be omitted. The case of real photon emission is special, because the EOM operators can generate nonzero contact terms in correlation functions with the EM current. The BG field technique provides an important simplification, since no contact terms are present whatsoever,

but instead the EOM are modified by the presence of the BG field. As a simple example, consider the following local operator with two covariant derivatives:

$$\mathcal{O}_{\mu\nu} = \bar{q}\gamma_\mu\gamma_\alpha \vec{D}_\alpha \vec{D}_\nu q. \quad (2.13)$$

For hadronic matrix-elements, one can use the identity $[D_\alpha, D_\nu] = -igG_{\alpha\nu}$ to write

$$\langle h'|\mathcal{O}_{\mu\nu}|h\rangle = -i\langle h'|\bar{q}\gamma_\mu\gamma_\alpha gG_{\alpha\nu}q|h\rangle + \langle h'|\bar{q}\gamma_\mu \vec{D}_\nu \vec{\not{D}} q|h\rangle \quad (2.14)$$

and neglect the last term. For photons, one either has to take into account the EOM operator explicitly, or use the BG field formalism, in which case there is an additional term in the commutator: $[D_\alpha, D_\nu] = -igG_{\alpha\nu} - ieF_{\alpha\nu}$. We get

$$\langle 0|\mathcal{O}_{\mu\nu}|0\rangle_F = -i\langle 0|\bar{q}\gamma_\mu\gamma_\alpha gG_{\alpha\nu}q|0\rangle_F - ie_q F_{\mu\nu}\langle \bar{q}q\rangle + \langle 0|\bar{q}\gamma_\mu \vec{D}_\nu \vec{\not{D}} q|0\rangle_F. \quad (2.15)$$

The third term on the r.h.s. is again zero by virtue of the EOM, $\not{D}q = 0$, but there is an additional contribution containing the photon field (and the quark condensate $\langle \bar{q}q\rangle$), which illustrates the generic feature of the BG field formalism, the appearance of new operators containing the EM field-strength tensor.

2.2 Light-cone expansion

After these general remarks, we proceed to the explicit construction of the light-cone expansion of quark-antiquark operators in the BG field. We assume that the quark and the antiquark are separated by the distance x_μ and take the limit $x^2 \rightarrow 0$ keeping the scalar product $(q \cdot x)$, $q^2 = 0$, fixed. As discussed in detail in [17, 18], the light-cone expansion of a generic nonlocal operator produces two sequences of terms that are singular and analytic on the light-cone, respectively:²

$$\langle 0|\bar{q}(x)\Gamma[x, y]_F q(y)|0\rangle_F = \langle 0|\bar{q}(x)\Gamma[x, y]_F q(y)|0\rangle_F^{\text{singular}} + \langle 0|\bar{q}(x)\Gamma[x, y]_F q(y)|0\rangle_F^{\text{analytic}}. \quad (2.16)$$

The singular contributions correspond to small distances, i.e. to the coefficient function in the OPE expansion, and the analytic contributions induce the corresponding operator matrix-elements. The separation between singular and analytic contributions involves, as usual, a factorization scale μ_F . In the case at hand, the singular contributions are given by

$$\langle 0|\bar{q}(x)\Gamma[x, y]_F q(y)|0\rangle_F^{\text{singular}} = -\text{Tr} [\Gamma[x, y]_F S(y-x)], \quad (2.17)$$

where the trace is taken over both spinor and colour indices and $S(y-x)$ is the quark propagator in the BG field (2.5). As for the analytic contributions, they can be *parametrized* in terms of nonperturbative photon DAs of increasing twist. To leading order in the QCD coupling and for massless quarks, one obtains for the chiral-even operators

$$\langle 0|\bar{q}(x)\gamma_\mu[x, -x]_F q(-x)|0\rangle_F = \int_0^1 du \, x^\nu e_q F_{\nu\mu}(-\xi x) \left\{ -\frac{N_c}{4\pi^2 x^2} \xi + \mathbb{G}^{(v)}(u) + \mathcal{O}(x^2) \right\},$$

²Cf. in particular Eqs. (3.7)–(3.11) in [18].

$$\langle 0 | \bar{q}(x) \gamma_\mu \gamma_5 [x, -x]_F q(-x) | 0 \rangle_F = i \int_0^1 du \, x^\nu e_q \tilde{F}_{\nu\mu}(-\xi x) \left\{ \frac{N_c}{4\pi^2 x^2} + \mathbb{G}^{(a)}(u) + \mathcal{O}(x^2) \right\} \quad (2.18)$$

where³

$$\xi = 2u - 1$$

and $\mathbb{G}^{(v)}(u)$, $\mathbb{G}^{(a)}(u)$ are nonperturbative functions that depend on x^2 at most logarithmically and will be determined later. Here and below we use the standard notation $\tilde{F}_{\alpha\beta} = (1/2)\epsilon_{\alpha\beta\mu\nu} F^{\mu\nu}$ and the conventions for the ϵ tensor and γ_5 matrix as defined in [19]. Counting dimensions, one finds that the perturbative contribution in (2.18) is twist-1, and the nonperturbative corrections are twist-3. Note that this dimensional twist-counting refers to the “dynamical” twist of a matrix-element, as opposed to the “geometric” twist of a (local) operator. We will discuss this in more detail in Sec. 4.1.

For chiral-odd operators the perturbative (singular) contribution vanishes identically for massless quarks. One then has [13]

$$\begin{aligned} \langle 0 | \bar{q}(x) \sigma_{\mu\nu} [x, -x]_F q(-x) | 0 \rangle_F &= e_q \langle \bar{q}q \rangle \int_0^1 du \, F_{\mu\nu}(-\xi x) \left\{ \chi \phi_\gamma(u, \mu) + \frac{x^2}{4} \mathbb{A}(u) \right\} \\ &\quad + \frac{1}{2} e_q \langle \bar{q}q \rangle \int_0^1 du \, x^\rho \{ x^\nu F_{\mu\rho} - x_\mu F_{\nu\rho} \} (-\xi x) \mathbb{B}(u) + \dots, \\ \langle 0 | \bar{q}(x) [x, -x]_F q(-x) | 0 \rangle_F &= 0. \end{aligned} \quad (2.19)$$

The function $\phi_\gamma(u, \mu)$ is the (nonperturbative) photon DA of leading twist-2, while the terms in $\mathbb{A}(u)$ and $\mathbb{B}(u)$ correspond to twist-4 corrections. They will be discussed in detail in the following sections. Note that the vacuum expectation value of the scalar operator does not contain any contribution linear in the BG field and thus vanishes identically.

For the strange quark, and also considering possible lattice calculations of DAs, it can be interesting to keep the lowest order term in the quark mass m_q . To leading-twist accuracy one then has

$$\begin{aligned} \langle 0 | \bar{q}(x) \sigma_{\mu\nu} [x, -x]_F q(-x) | 0 \rangle_F &= \int_0^1 du \, e_q F_{\mu\nu}(-\xi x) \left\{ -\frac{N_c m_q}{4\pi^2} \left[\ln[x^2 \mu^2 u(1-u)] + \text{const} \right] \right. \\ &\quad \left. + \chi \langle \bar{q}q \rangle \phi_\gamma(u, \mu) \right\} + \dots \end{aligned} \quad (2.20)$$

(cf. App. A in [11]). Note that in this case there is a weaker, logarithmic singularity on the light-cone. The calculation of the constant under the logarithm requires a careful matching with the UV-divergent contribution of the quark loop to the photon DA [11].

³The definition of the integration variable u is a convention. It is chosen such that in the case of an *ingoing* photon u corresponds to the momentum fraction carried by the quark.

It is instructive to verify that the perturbative (singular) contributions to (2.18) indeed correspond to the perturbative wave-function of the real photon in the infinite-momentum frame. To this end choose $F_{\mu\nu}$ as a plane-wave, (2.4), and the coordinate system such that⁴ $q = (q^+, 0, 0)$, $x = (0, x_-, \vec{r})$, $x^2 = -\vec{r}^2$, where \vec{r} is a two-dimensional vector in the transverse plane. For the leading-twist “plus” component, assuming physical (transverse) photon polarization, one obtains

$$\langle 0 | \bar{q}(x) \gamma_+ \frac{1 \pm \gamma_5}{2} [x, -x]_F q(-x) | 0 \rangle_F = -\frac{i N_c e_q}{8\pi^2 \mathbf{r}^2} q_+ \int_0^1 du e^{i\xi q_+ x_-} \left[(e^{(\lambda)} \cdot \mathbf{r})(2u - 1) \pm i \epsilon_{ik} \mathbf{r}_i e_k^{(\lambda)} \right], \quad (2.21)$$

where $\epsilon_{ik} = \epsilon_{ik+-}$ is the two-dimensional antisymmetric tensor in the transverse plane, $i, k = 1, 2$, which is the expected result. Indeed, the photon wave-function can be defined as

$$\begin{aligned} \psi_h^\lambda(u, \mathbf{r}) &= \int \frac{dx_-}{\pi} e^{-ix_- q_+ (2u-1)} \langle 0 | \bar{q}(x) \gamma_+ \frac{1 + h\gamma_5}{2} [x, -x]_F q(-x) | 0 \rangle_F \Big|_{x_+ = 0, x_\perp = \mathbf{r}} \\ &= -\frac{i N_c e_q}{8\pi^2 \mathbf{r}^2} \left\{ (e^{(\lambda)} \cdot \mathbf{r})(2u - 1) + h i \epsilon_{ik} \mathbf{r}_i e_k^{(\lambda)} \right\} \end{aligned} \quad (2.22)$$

where $h = \pm 1$ corresponds to the quark helicity. Choosing the photon polarisation vector as $e^{(\pm)} = (0, 1, \pm i, 0)/\sqrt{2}$, one obtains the familiar expressions given in, for instance, [20, 21]⁵.

3 The Leading-Twist Photon Distribution Amplitude

We proceed with the detailed study of the leading-twist DA of the photon. Like for a transversely polarized vector meson [22, 23], the leading-twist contribution corresponds to the chiral-odd Lorentz-structure. Following [11] we define the DA as the vacuum expectation value (VEV) of the nonlocal quark-antiquark operator with light-like separations,

$$\langle 0 | \bar{q}(z) \sigma_{\alpha\beta} [z, -z]_F q(-z) | 0 \rangle_F = e_q \chi \langle \bar{q}q \rangle \int_0^1 du F_{\alpha\beta}(-\xi z) \phi_\gamma(u), \quad (3.1)$$

where $z^2 = 0$ and we have inserted the path-ordered gauge-factor which was implied, but not shown in (1.2). This definition is equivalent to

$$\begin{aligned} i \int d^4 y e^{-iqy} \langle 0 | T \{ j^\mu(y) \bar{q}(z) \sigma_{\alpha\beta} [z, -z] q(-z) \} | 0 \rangle &= i (q_\beta g_{\mu\alpha} - q_\alpha g_{\mu\beta}) \chi \langle \bar{q}q \rangle \int_0^1 du e^{i\xi qz} \phi_\gamma(u) \\ &+ \frac{z_\mu}{qz} [e^{iqz} - e^{-iqz}] \langle 0 | \bar{q}(z) \sigma_{\alpha\beta} [z, -z] q(-z) | 0 \rangle, \end{aligned} \quad (3.2)$$

⁴We define $a^\pm = (a^0 \pm a^3)/\sqrt{2}$ for any four-vector a .

⁵Up to a different normalization.

cf. (2.7) and (2.8). The first term on the r.h.s. of (3.2) corresponds to the VEV in the BG field of the plane-wave, (3.1), (2.4), and the second term is the contact term. In fact, in the above case the contact term vanishes: the VEV $\langle 0|\bar{q}(z)\sigma_{\alpha\beta}[z, -z]q(-z)|0\rangle$ does not include any external momentum and from z_α alone it is not possible to build an antisymmetric Lorentz-structure (in (α, β)).

Yet another form of the definition (3.2) is (cf. (2.9))

$$\langle 0|\bar{q}(z)\sigma_{\alpha\beta}[z, -z]q(-z)|\gamma^{(\lambda)}(q)\rangle = i e_q \chi \langle \bar{q}q \rangle \left(q_\beta e_\alpha^{(\lambda)} - q_\alpha e_\beta^{(\lambda)} \right) \int_0^1 du e^{i\xi qz} \phi_\gamma(u). \quad (3.3)$$

Here we assume that the physical photon polarization is transverse to the (q, z) plane. The three definitions (3.1), (3.2) and (3.3) are equivalent and can be useful in different contexts.

As always in quantum field theory, the extraction of the asymptotic behaviour $z^2 \rightarrow 0$ creates divergences that have to be regularized. As a result, both the magnetic susceptibility χ and the DA $\phi_\gamma(u)$ are scale-dependent. To leading-logarithmic accuracy:

$$\chi(\mu) = L^{(\gamma_0 - \gamma_{\bar{q}q})/b} \chi(\mu_0),$$

$$\phi_\gamma(u, \mu) = 6u\bar{u} \left[1 + \sum_{n=2,4,\dots}^{\infty} L^{(\gamma_n - \gamma_0)/b} \phi_n(\mu_0) C_n^{3/2}(u - \bar{u}) \right], \quad (3.4)$$

where $C_n^{3/2}(x)$ are Gegenbauer-polynomials, $b = 11/3N_c - 2/3n_f$, $L = \alpha_s(\mu)/\alpha_s(\mu_0)$, $\gamma_{\bar{q}q} = -3C_F$ is the anomalous dimension of the quark condensate and

$$\gamma_n = C_F \left[1 + 4 \sum_{j=2}^{n+1} \frac{1}{j} \right] \quad (3.5)$$

are the anomalous dimensions of the chiral-odd local operators of leading-twist [24].

Photon DAs are interesting mainly because of the fact that the magnetic susceptibility χ appears to be rather large. A crude estimate of its value can be obtained in the vector-dominance approximation [25]: consider the correlation function

$$i \int d^4y e^{-iqy} \langle 0|T\{j^\mu(y)\bar{q}(0)\sigma_{\alpha\beta}q(0)\}|0\rangle = i(q_\beta g_{\mu\alpha} - q_\alpha g_{\mu\beta})\chi(q^2, \mu) \langle \bar{q}q \rangle, \quad (3.6)$$

where in contrast to (3.2) the separation between the quarks is set zero and q^2 can be arbitrary, instead of $q^2 = 0$ for a real photon. From the normalization of the photon DA it follows that $\chi(\mu) \equiv \chi(q^2 = 0, \mu)$. Assuming that the correlation function is dominated by the contribution of the lowest-lying states ρ_0 and ω and using the standard definitions of couplings⁶ in the flavour-SU(2) limit

$$\langle 0|[\bar{q}\gamma_\mu q]^{I=1(0)}|\rho_0(\omega)\rangle = e_\mu^{(\lambda)} f_V m_V, \quad f_V = 215 \text{ MeV} [26],$$

$$\langle 0|[\bar{q}\sigma_{\alpha\beta} q]^{I=1(0)}|\rho_0(\omega)\rangle = i(e_\alpha^{(\lambda)} q_\beta - e_\beta^{(\lambda)} q_\alpha) f_V^\perp, \quad f_V^\perp = (160 \pm 10) \text{ MeV} [23], \quad (3.7)$$

⁶The currents with specific isospin are given by $[\bar{q}\Gamma q]^{I=1(0)} = (\bar{u}\Gamma u \mp \bar{d}\Gamma d)/\sqrt{2}$. m_V is the average mass of ω and $\rho(770)$.

one obtains

$$\langle \bar{q}q \rangle \chi(q^2) \simeq -\frac{m_V f_V f_V^\perp}{m_V^2 - q^2}, \quad (3.8)$$

and therefore

$$\chi_{\text{VDM}} \simeq -\frac{f_V f_V^\perp}{m_V \langle \bar{q}q \rangle} \simeq 2.7 \text{ GeV}^{-2}, \quad (3.9)$$

using the value $\langle \bar{q}q \rangle = -(250 \text{ MeV})^3$ (at 1 GeV)⁷. Alternatively, one can assume that the vector-dominance approximation (3.8) is valid for large Euclidian $q^2 \rightarrow -\infty$, in which limit the leading contribution to the correlation function (3.6) is given by the quark condensate:

$$\chi(q^2) \stackrel{q^2 \rightarrow -\infty}{\simeq} \frac{2}{-q^2}. \quad (3.10)$$

Comparing this expression with (3.8), one obtains an even simpler estimate [25]

$$\chi_{\text{LD}} \simeq \frac{2}{m_V^2} \simeq 3.3 \text{ GeV}^{-2}, \quad (3.11)$$

which is rather close to (3.9). Here the subscript “LD” stands for the so-called local-duality approximation.

A more sophisticated study including contributions of higher resonances to (3.8) on one side, and contributions of higher-dimensional operators to (3.11) on the other side, combined with the usual QCD sum rule techniques to match between the two representations in the most efficient way, was done in [27, 28]. The result of this analysis,

$$\chi_{\text{SR}}(1 \text{ GeV}) \simeq 4.4 \text{ GeV}^{-2}, \quad (3.12)$$

was used in numerous QCD sum rule calculations of EM hadron properties over the last 15 years. In App. B we give an update of the analysis of [27, 28], using new data for the resonances, the most recent value of α_s and including $O(\alpha_s)$ radiative corrections to the sum rule. We obtain

$$\begin{aligned} \chi_{\text{SR}}(1 \text{ GeV}) &\simeq (3.15 \pm 0.3) \text{ GeV}^{-2}, \\ f_\gamma^\perp(1 \text{ GeV}) &\equiv \chi \langle \bar{q}q \rangle = -(50 \pm 15) \text{ MeV}, \end{aligned} \quad (3.13)$$

which is smaller compared to the older estimate, mostly due to the large negative radiative correction.

The contribution of the photon DA (twist-2) to a generic hard exclusive process is, compared to the perturbative contribution (twist-1), suppressed by one power of the hard scale Q and is of order

$$\frac{1}{Q} \chi \frac{(2\pi)^2 |\langle \bar{q}q \rangle|}{N_c} \approx \frac{650 \text{ MeV}}{Q}. \quad (3.14)$$

This relatively large mass scale is due to the large magnetic susceptibility and to the well-known fact the the quark condensate contribution to physical observables is enhanced by

⁷The sign of χ depends on the sign convention used for the EM coupling in (2.1).

a factor $(2\pi)^2$ compared to the perturbative contributions because of a different structure of the phase-space integrals, cf. Eqs. (2.18) and (2.19).

The available information on the shape of the photon DA is more controversial. At large virtualities, the shape is uniquely determined by the renormalization properties of the relevant operators, cf. (3.4). As the anomalous dimensions in (3.5) rise with n , one obtains, for very large scales μ , the asymptotic photon DA

$$\phi_\gamma^{\text{asy}}(u) = 6u(1 - u), \quad (3.15)$$

which is the same as for π, ρ, \dots mesons. In Ref. [11] arguments were given suggesting that at smaller virtualities of order 1 GeV the photon DA is still close to the asymptotic form. The physical picture behind the argumentation in [11] was that corrections to the asymptotic DA are nonperturbative in nature, and are expected to be washed out when one sums over contributions of different hadron resonances $\rho, \rho', \rho'', \dots$. This qualitative conclusion was supported by the calculation of the second Gegenbauer-coefficient ϕ_2 at the scale 1 GeV using QCD sum rules, yielding a small number. Unfortunately, this estimate appears to be very sensitive to the choice of input parameters, see App. B. In addition, there has been increasing evidence in the last years indicating that “standard” QCD sum rule estimates of corrections to asymptotic DAs are not reliable. We therefore believe that the existing evidence in favour of the asymptotic DA of the photon at low scales is only qualitative.

Another calculation of the photon DA was done in [14], using the instanton-model of the QCD vacuum. This approach neglects altogether perturbative gluon radiation which generates the asymptotic DA, and the results correspond to a very low normalization point of order of the characteristic instanton-size $\rho_I \sim 600$ MeV. The resulting photon DA can to a good accuracy be represented by a simple expression

$$\phi_\gamma^{\text{IM}}(u, \mu = 600 \text{ MeV}) = \frac{1}{a - 1} [a - 6u(1 - u)] \quad (3.16)$$

with $a \simeq 7.58$.⁸ In the instanton-model the shape of the pion and the photon DA turn out to be very different. The reason for this is that for the pion there exists an additional (to perturbative radiation) mechanism that suppresses the DA at the end-points [14]. Roughly speaking, this effect is due to a nonvanishing size of a constituent quark when it is probed by the axial current, which is required quite generally by chiral symmetry breaking. For the vector current (photon) the constituent quarks appear to be pointlike, and the suppression is not required. As a result, the pion distribution is predicted in this model to be close to the asymptotic one already at very low scales, while the photon DA is almost flat and does not vanish at the end-points, see Eq. (3.16) and Fig. 2. From our point of view this evidence is not conclusive either, since the observed end-point behaviour corresponds to contributions of large invariant masses, and it is not clear where inclusion of them in the instanton-model calculation is legitimate. By its physical meaning, the distribution amplitude corresponds to the contributions of low momenta and we expect that an asymptotic-like shape would be recovered if the calculations were made with an

⁸V.M.B. thanks M. Polyakov and C. Weiss for providing this parametrization.

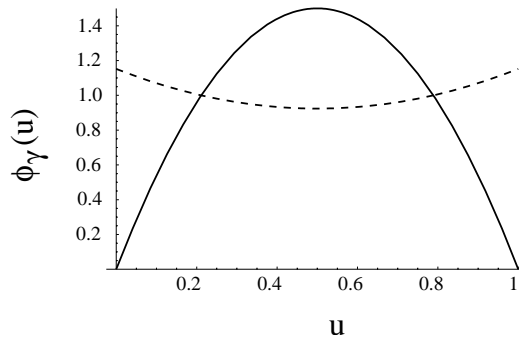


Figure 2: The chiral-odd photon DA of leading twist. The two curves correspond to the asymptotic DA (3.15) (solid) and the instanton model at the low normalization point 0.6 GeV (3.16) (dashed).

explicit ultraviolet cutoff of order of the instanton-size. We believe that this question will eventually be decided by lattice calculations and/or experiments on hard exclusive dijet photoproduction in which the shape of the photon DA can be measured [29].

For the strange quark, one might want to take into account the quark mass corrections, in which case there is the complication that the separation of singular and analytic contributions on the light-cone involves a factorization scale, cf. (2.20). For completeness, we quote here the corresponding result from [11]:

$$\begin{aligned} \langle 0 | \bar{s}(x) \sigma_{\alpha\beta} [x, -x] s(-x) | \gamma^{(\lambda)}(q) \rangle &= i e_s \left(q_\beta e_\alpha^{(\lambda)} - q_\alpha e_\beta^{(\lambda)} \right) \times \\ &\times \int_0^1 du e^{i\xi qx} \left\{ \left(\chi_s \langle \bar{s}s \rangle - \frac{27m_s}{g_\phi^2} \right) \phi_\gamma(u) - \frac{3m_s}{4\pi^2} \left[\ln(-x^2 w_0 u \bar{u}) - \frac{36\pi^2}{g_\phi^2} + 2\gamma_E \right] \right\} \end{aligned} \quad (3.17)$$

The quantity $w_0 = 2 \text{ GeV}^2$ is the continuum threshold in the classical QCD sum rule for the ϕ -meson [30] and $4\pi^2/g_\phi^2 \simeq 11.7$ is the ϕ -meson coupling (squared) to the EM current. By definition, the magnetic susceptibility of the quark condensate does not include perturbative contributions of the quark loop $\sim m_s$.

4 Distribution Amplitudes of Higher Twist

4.1 General classification

The aim of this section is to give a general classification of photon DAs up to twist-4. As already stated in Sec. 1, we define DAs as (renormalized) matrix-elements, in the EM BG field, of nonlocal gauge-invariant operators at strictly light-like separations. For brevity, starting from this section we will not show path-ordered gauge-factors between the fields, they are always implied. We distinguish between matrix-elements of operators with an odd number of Dirac γ -matrices, which we term chiral-even, and operators with an even number of γ -matrices, which we refer to as chiral-odd.

The twist-counting employed in this classification is based on counting of powers of the photon momentum in the infinite-momentum frame, cf. the discussion in [31]. To

Twist	(μ)	$\bar{q}\gamma_\mu q$	$\bar{q}\gamma_\mu\gamma_5 q$	$(\mu\nu)$	$\bar{q}\sigma_{\mu\nu} q$
2				$+$	$\perp \quad \phi_\gamma$
3	\perp	$\psi^{(v)}$	$\psi^{(a)}$		
4				$\perp -$	h_γ

Table 1: Identification of two-particle DAs with projections onto different light-cone components of the nonlocal operators. For example, \perp refers to $\bar{q}\gamma_\perp\gamma_5 q$.

Twist	$(\mu\nu\alpha)$	$\bar{q}\widetilde{G}_{\mu\nu}\gamma_\alpha\gamma_5q$	$\bar{q}G_{\mu\nu}\gamma_\alpha q$	$(\mu\nu\alpha\beta)$	$\bar{q}G_{\mu\nu}\sigma_{\alpha\beta}q$	$(\mu\nu)$	$\bar{q}G_{\mu\nu}q$	$\bar{q}\widetilde{G}_{\mu\nu}\gamma_5q$
3	$+\perp+$	\mathcal{A}	\mathcal{V}					
4				$\perp\perp+\perp$	T_1	$+\perp$	S	\widetilde{S}
				$+\perp\perp\perp$	T_2			
				$+-+\perp$	T_3			
				$+\perp+-$	T_4			

Table 2: Same as Tab. 1 for three-particle DAs. $+\perp\perp$ refers to $\bar{q}\tilde{G}_{+\perp}\gamma_\perp\gamma_5 q$ etc.

explain this, take a plane-wave configuration for $F_{\mu\nu}$ and choose the frame of reference such that $q_\mu = (q_+, 0, \mathbf{0}_\perp)$, $z_\mu = (0, z_-, \mathbf{0}_\perp)$ with $q_+ \rightarrow \infty$. The twist-expansion corresponds to counting powers of the large momentum q_+ , taking into account that the photon's transverse polarization vector is of order $e_\mu^{(\lambda)} \sim (q_+)^0$ and $z_\mu \sim (q_+)^{-1}$. Matrix-elements of nonlocal operators typically involve several Lorentz-structures which can be separated by suitable projection operators. In this way, the leading-twist structures correspond to the maximum number of “plus” components. Each replacement of a “plus” projection by a transverse one increases the twist by one unit, and each “minus” projection adds two units of twist.

The complete classification of the relevant two- and three-particle DAs and their relation to specific light-cone projections of the matrix-elements is given in Tabs. 1 and 2, respectively. As is well known, “minus” components of quark-field operators do not describe independent partonic degrees of freedom, but can be expressed in terms of multiparton DAs. The explicit definitions and relations between various distributions will be worked out in the next sections.

Note that the classification of photon DAs is very similar to the classification of the DAs of the transversely polarized ρ -meson discussed in [31, 32]. For the twist-3 distributions considered in the next section the analogy is in fact one-to-one. For twist-4 distributions the situation is somewhat more complicated because of the different structure of the QCD EOM in the presence of the BG field.

4.2 Twist-3 distribution amplitudes

The twist-3 DAs of a real photon are related to matrix-elements of chiral-even operators. Since the vacuum expectation value of the vector current in the BG EM field vanishes by the EOM, $\langle 0|\bar{q}\gamma_\mu q|0\rangle_F = 0$, the relevant local operator with lowest dimension has to include the gluon field:

$$\langle 0|\bar{q}g\tilde{G}_{\mu\nu}\gamma_\alpha\gamma_5 q|0\rangle_F = e_q f_{3\gamma} D_\alpha F_{\mu\nu}, \quad (4.1)$$

where D_α is the covariant derivative. The nonperturbative constant $f_{3\gamma}$ has dimension GeV^2 and provides a natural mass-scale for the twist-3 DAs. There are two two-particle DAs which we define as (cf. Tab. 1)

$$\begin{aligned} \langle 0|\bar{q}(z)\gamma_\mu q(-z)|0\rangle_F &= e_q f_{3\gamma} \int_0^1 du \bar{\psi}^{(v)}(u, \mu) F_{z\mu}(-\xi z), \\ \langle 0|\bar{q}(z)\gamma_\mu\gamma_5 q(-z)|0\rangle_F &= e_q f_{3\gamma} \frac{i}{2} \int_0^1 du \psi^{(a)}(u, \mu) \tilde{F}_{z\mu}(-\xi z), \end{aligned} \quad (4.2)$$

where

$$F_{z\mu} = F_{\rho\mu} z^\rho.$$

We will use similar shorthand notations in what follows. The normalization of the DAs is such that $\int_0^1 du \bar{\psi}^{(v)}(u) = \int_0^1 du \psi^{(a)}(u) = 0$. Note that the regularization (and renormalization) removes the singular $1/z^2$ contributions, so that the matrix-elements in (4.2) correspond to the analytic parts of the amplitudes in the light-cone limit, in the sense of the separation in Eq. (2.16). Comparing (4.2) with (2.18) we find

$$\begin{aligned} \mathbb{G}^{(v)}(u) &= f_{3\gamma} \bar{\psi}^{(v)}(u, \mu \sim 1/|x|), \\ \mathbb{G}^{(a)}(u) &= \frac{1}{2} f_{3\gamma} \psi^{(a)}(u, \mu \sim 1/|x|). \end{aligned} \quad (4.3)$$

In addition, there exist two three-particle chiral-even twist-3 distributions:

$$\begin{aligned} \langle 0|\bar{q}(z)g\tilde{G}_{\mu\nu}(vz)\gamma_\alpha\gamma_5 q(-z)|0\rangle_F &= e_q f_{3\gamma} \int \mathcal{D}\underline{\alpha} \mathcal{A}(\underline{\alpha}) D_\alpha F_{\mu\nu}(\alpha_v z), \\ \langle 0|\bar{q}(z)gG_{\mu\nu}(vz)i\gamma_\alpha q(-z)|0\rangle_F &= e_q f_{3\gamma} \int \mathcal{D}\underline{\alpha} \mathcal{V}(\underline{\alpha}) D_\alpha F_{\mu\nu}(\alpha_v z), \end{aligned} \quad (4.4)$$

where $\alpha_v = \alpha_{\bar{q}} - \alpha_q + v\alpha_g$, and $\underline{\alpha}$ is the set of the three momentum fractions $\underline{\alpha} = \{\alpha_q, \alpha_{\bar{q}}, \alpha_g\}$. The integration measure is defined as

$$\int \mathcal{D}\underline{\alpha} \equiv \int_0^1 d\alpha_q \int_0^1 d\alpha_{\bar{q}} \int_0^1 d\alpha_g \delta\left(1 - \sum \alpha_i\right). \quad (4.5)$$

Alternatively, the same distributions can be defined as matrix-elements between the ingoing photon state and the vacuum, and in this form they are very similar to the definitions adopted in [31, 32] for the transversely polarized ρ -meson, apart from a different overall

normalization. For two-particle distributions:

$$\begin{aligned}\langle 0|\bar{q}(z)\gamma_\mu q(-z)|\gamma^{(\lambda)}(q)\rangle &= e_q f_{3\gamma} e_{\perp\mu}^{(\lambda)} \int_0^1 du e^{i\xi qz} \psi^{(v)}(u, \mu), \\ \langle 0|\bar{q}(z)\gamma_\mu \gamma_5 q(-z)|\gamma^{(\lambda)}(q)\rangle &= \frac{1}{2} e_q f_{3\gamma} \epsilon_{\mu\nu qz} e_{\perp\nu}^{(\lambda)} \int_0^1 du e^{i\xi qz} \psi^{(a)}(u, \mu).\end{aligned}\quad (4.6)$$

The functions $\bar{\psi}^{(v)}(u, \mu)$ and $\psi^{(v)}(u, \mu)$ are related by integration by parts:

$$\bar{\psi}^{(v)}(u, \mu) = 2 \int_0^u d\alpha \psi^{(v)}(\alpha, \mu). \quad (4.7)$$

For the three-particle distributions one has

$$\begin{aligned}\langle 0|\bar{q}(z)g\tilde{G}_{\mu\nu}(vz)\gamma_\alpha \gamma_5 q(-z)|\gamma^{(\lambda)}(q)\rangle &= e_q f_{3\gamma} q_\alpha [q_\nu e_{\perp\mu}^{(\lambda)} - q_\mu e_{\perp\nu}^{(\lambda)}] \int \mathcal{D}\underline{\alpha} \mathcal{A}(\underline{\alpha}) e^{-iqz\alpha_v}, \\ \langle 0|\bar{q}(z)gG_{\mu\nu}(vz)i\gamma_\alpha q(-z)|\gamma^{(\lambda)}(q)\rangle &= e_q f_{3\gamma} q_\alpha [q_\nu e_{\perp\mu}^{(\lambda)} - q_\mu e_{\perp\nu}^{(\lambda)}] \int \mathcal{D}\underline{\alpha} \mathcal{V}(\underline{\alpha}) e^{-iqz\alpha_v}.\end{aligned}\quad (4.8)$$

Like the corresponding ρ -meson DAs, the two-particle distributions $\psi^{(v)}(u)$ and $\psi^{(a)}(u)$ are not independent and can be related to three-particle distributions using the QCD EOM. A closer inspection shows that all corrections to the EOM in the BG field vanish in the chiral-even case, so that these relations remain intact.⁹ As result, the photon DAs can be read off the corresponding expressions for the ρ -meson [32], where one must omit the so-called Wandzura-Wilczek parts that are due to the nonzero meson mass. To next-to-leading order in the conformal expansion, cf. Sec. 4.3, we obtain

$$\begin{aligned}\mathcal{V}(\underline{\alpha}) &= 540 \omega_\gamma^V (\alpha_q - \alpha_{\bar{q}}) \alpha_q \alpha_{\bar{q}} \alpha_g^2, \\ \mathcal{A}(\underline{\alpha}) &= 360 \alpha_q \alpha_{\bar{q}} \alpha_g^2 \left[1 + \omega_\gamma^A \frac{1}{2} (7\alpha_g - 3) \right], \\ \psi^{(v)}(u) &= 5(3\xi^2 - 1) + \frac{3}{64} (15\omega_\gamma^V - 5\omega_\gamma^A) (3 - 30\xi^2 + 35\xi^4), \\ \psi^{(a)}(u) &= (1 - \xi^2) (5\xi^2 - 1) \frac{5}{2} \left(1 + \frac{9}{16} \omega_\gamma^V - \frac{3}{16} \omega_\gamma^A \right), \\ \bar{\psi}^{(v)}(u) &= -20u\bar{u}\xi + \frac{15}{16} (\omega_\gamma^A - 3\omega_\gamma^V) u\bar{u}\xi (7\xi^2 - 3),\end{aligned}\quad (4.9)$$

where the parameters ω_γ^A and ω_γ^V correspond to matrix-elements of local operators of dimension 6, see [32] and App. B.

In vector-dominance approximation one finds

$$\begin{aligned}f_{3\gamma}^{\text{VDM}} &= -f_\rho^2 \zeta_3 \simeq -(4 \pm 2) \cdot 10^{-3} \text{ GeV}^2, \\ \omega_\gamma^{V, \text{VDM}} &= \omega_\rho^V \simeq 3.8 \pm 1.8, \\ \omega_\gamma^{A, \text{VDM}} &= \omega_\rho^A \simeq -2.1 \pm 1.0,\end{aligned}\quad (4.10)$$

⁹This is in contrast to the chiral-odd ones, to be discussed in the following subsection, where one VEV is nontrivial, and induces a nonzero conformal expansion of an exactly vanishing DA.

where $f_\rho \zeta_3 \simeq (20 \pm 10)$ MeV (see App. B) is the ρ -meson coupling to the quark-antiquark-gluon current in (4.1). Our result for this coupling is by a factor 3 larger than an old estimate in obtained in Ref. [33], and quoted in [31, 32]. We have estimated corrections to the vector-dominance approximation for $f_{3\gamma}$ using QCD sum rules, see App. B, with the result that these corrections are smaller than the (large) error bars for the ρ -meson contribution. We, therefore, accept the vector-dominance approximation for the present.

The scale-dependence of the twist-3 parameters is given by, cf. Ref. [31], ($C_A = N_c$):

$$f_{3\gamma}(\mu^2) = L^{\gamma^f/b} f_{3\gamma}(\mu_0^2), \quad \gamma^f = -\frac{1}{3}C_F + 3C_A, \quad (4.11)$$

and

$$\begin{aligned} \left(\frac{\omega_\gamma^V - \omega_\gamma^A}{\omega_\gamma^V + \omega_\gamma^A} \right)^{\mu^2} &= L^{\Gamma^\omega/b} \left(\frac{\omega_\gamma^V - \omega_\gamma^A}{\omega_\gamma^V + \omega_\gamma^A} \right)^{\mu_0^2}, \\ \Gamma^\omega &= \begin{pmatrix} 3C_F - \frac{2}{3}C_A & \frac{2}{3}C_F - \frac{2}{3}C_A \\ \frac{5}{3}C_F - \frac{4}{3}C_A & \frac{1}{2}C_F + C_A \end{pmatrix}. \end{aligned} \quad (4.12)$$

4.3 Twist-4 distribution amplitudes

4.3.1 Definitions

Twist-4 DAs, which are all chiral-odd, are more involved and require a detailed discussion. The general parametrization of the nonlocal quark-antiquark operator on the light-cone involves two invariant functions:

$$\begin{aligned} \langle 0 | \bar{q}(z) \sigma_{\mu\nu} q(-z) | 0 \rangle_F &= e_q \chi \langle \bar{q}q \rangle \int_0^1 du \phi_\gamma(u, \mu) F_{\mu\nu}(-\xi z) \\ &+ \frac{1}{2} e_q \langle \bar{q}q \rangle \int_0^1 du \bar{h}_\gamma(u, \mu) \{ z_\nu F_{\mu z} - z_\mu F_{\nu z} \} (-\xi z), \\ \langle 0 | \bar{q}(z) q(-z) | 0 \rangle_F &= 0. \end{aligned} \quad (4.13)$$

Here $\phi_\gamma(u)$ is the leading-twist distribution discussed in Sec. 3 and $\bar{h}_\gamma(u, \mu)$ is a new DA of twist-4, cf. Tab. 1. Note that the third possible Lorentz-structure $z^\rho \{ D^\nu F_{\mu\rho} - D_\mu F_{\nu\rho} \}$ is not independent and can be eliminated using the QED Bianchi-identity.

The three-particle DAs are more numerous and can be defined as:

$$\begin{aligned} \langle 0 | \bar{q}(z) g G_{\mu\nu}(vz) q(-z) | 0 \rangle_F &= e_q \langle \bar{q}q \rangle \int \mathcal{D}\underline{\alpha} S(\underline{\alpha}) F_{\mu\nu}(\alpha_v z), \\ \langle 0 | \bar{q}(z) g \tilde{G}_{\mu\nu}(vz) i\gamma_5 q(-z) | 0 \rangle_F &= e_q \langle \bar{q}q \rangle \int \mathcal{D}\underline{\alpha} \tilde{S}(\underline{\alpha}) F_{\mu\nu}(\alpha_v z) \end{aligned} \quad (4.14)$$

and

$$\langle 0 | \bar{q}(z) \sigma_{\alpha\beta} g G_{\mu\nu}(vz) q(-z) | 0 \rangle_F =$$

$$\begin{aligned}
&= ie_q \langle \bar{q}q \rangle [g_{\mu\rho} \{g_{\alpha\nu}^\perp D_\beta - g_{\beta\nu}^\perp D_\alpha\} - (\mu \leftrightarrow \nu)] \int \mathcal{D}\underline{\alpha} \bar{T}_1(\underline{\alpha}) F_{\rho z}(\alpha_v z) \\
&+ ie_q \langle \bar{q}q \rangle [g_{\alpha\rho} \{g_{\beta\mu}^\perp D_\nu - g_{\beta\nu}^\perp D_\mu\} - (\alpha \leftrightarrow \beta)] \int \mathcal{D}\underline{\alpha} \bar{T}_2(\underline{\alpha}) F_{\rho z}(\alpha_v z) \\
&+ ie_q \langle \bar{q}q \rangle [D_\mu z_\nu - D_\nu z_\mu] \int \mathcal{D}\underline{\alpha} \bar{T}_3(\underline{\alpha}) F_{\alpha\beta}(\alpha_v z) \\
&+ ie_q \langle \bar{q}q \rangle [D_\alpha z_\beta - D_\beta z_\alpha] \int \mathcal{D}\underline{\alpha} \bar{T}_4(\underline{\alpha}) F_{\mu\nu}(\alpha_v z), \tag{4.15}
\end{aligned}$$

see Tab. 2 for the identification of the relevant light-cone projections.

In addition, we have to consider also quark-antiquark-photon operators:

$$\langle 0 | \bar{q}(z) e_q F_{\mu\nu}(vz) q(-z) | 0 \rangle_F = e_q \langle \bar{q}q \rangle F_{\mu\nu}(vz) \tag{4.16}$$

and

$$\langle 0 | \bar{q}(z) \sigma_{\alpha\beta} e_q F_{\mu\nu}(vz) q(-z) | 0 \rangle_F = 0. \tag{4.17}$$

It is convenient to rewrite these expressions introducing auxiliary three-particle DAs:

$$\langle 0 | \bar{q}(z) e_q F_{\mu\nu}(vz) q(-z) | 0 \rangle_F = e_q \langle \bar{q}q \rangle \int \mathcal{D}\underline{\alpha} S_\gamma(\underline{\alpha}) F_{\mu\nu}(\alpha_v z), \tag{4.18}$$

$$\langle 0 | \bar{q}(z) \sigma_{\alpha\beta} e_q F_{\mu\nu}(vz) q(-z) | 0 \rangle_F = ie_q \langle \bar{q}q \rangle [D_\alpha z_\beta - D_\beta z_\alpha] \int \mathcal{D}\underline{\alpha} \bar{T}_4^\gamma(\underline{\alpha}) F_{\mu\nu}(\alpha_v z) + \dots \tag{4.19}$$

with

$$S_\gamma(\underline{\alpha}) = \delta(\alpha_{\bar{q}}) \delta(\alpha_q), \quad \bar{T}_4^\gamma(\underline{\alpha}) = 0. \tag{4.20}$$

As explained in the next subsection, we need the conformal expansion of $S_\gamma(\underline{\alpha})$ and $\bar{T}_4^\gamma(\underline{\alpha})$, which turn out to have nonzero coefficients.

In some applications it can be advantageous to rewrite the definitions (4.13) to (4.19) for the particular case of a photon in the initial state. For two-particle DAs we obtain

$$\begin{aligned}
\langle 0 | \bar{q}(z) \sigma_{\alpha\beta} q(-z) | \gamma^{(\lambda)}(q) \rangle &= ie_q \chi \langle \bar{q}q \rangle \left(q_\beta e_\alpha^{(\lambda)} - q_\alpha e_\beta^{(\lambda)} \right) \int_0^1 du e^{i\xi qz} \phi_\gamma(u, \mu) \\
&+ \frac{1}{2} ie_q \frac{\langle \bar{q}q \rangle}{qz} \left(z_\beta e_\alpha^{(\lambda)} - z_\alpha e_\beta^{(\lambda)} \right) \int_0^1 du e^{i\xi qz} h_\gamma(u, \mu). \tag{4.21}
\end{aligned}$$

It is easy to see that

$$\bar{h}_\gamma(u) = -4 \int_0^u d\alpha (u - \alpha) h_\gamma(\alpha, \mu). \tag{4.22}$$

C-parity implies the symmetry $\bar{h}_\gamma(u) = -\bar{h}_\gamma(1 - u)$ and $h_\gamma(u) = h_\gamma(1 - u)$.

For three-particle distributions the corresponding definitions read

$$\begin{aligned}\langle 0|\bar{q}(z)gG_{\mu\nu}(vz)q(-z)|\gamma^{(\lambda)}(q)\rangle &= ie_q\langle\bar{q}q\rangle[q_\nu e_{\perp\mu}^{(\lambda)} - q_\mu e_{\perp\nu}^{(\lambda)}] \int \mathcal{D}\underline{\alpha} S(\underline{\alpha}) e^{-i(qz)\alpha_v}, \\ \langle 0|\bar{q}(z)g\tilde{G}_{\mu\nu}(vz)i\gamma_5 q(-z)|\gamma^{(\lambda)}(q)\rangle &= ie_q\langle\bar{q}q\rangle[q_\nu e_{\perp\mu}^{(\lambda)} - q_\mu e_{\perp\nu}^{(\lambda)}] \int \mathcal{D}\underline{\alpha} \tilde{S}(\underline{\alpha}) e^{-i(qz)\alpha_v},\end{aligned}\quad (4.23)$$

and

$$\begin{aligned}\langle 0|\bar{q}(z)\sigma_{\alpha\beta}gG_{\mu\nu}(vz)q(-z)|\gamma^{(\lambda)}(q)\rangle &= \\ &e_q\langle\bar{q}q\rangle[q_\alpha e_{\perp\mu}^{(\lambda)}g_{\beta\nu}^\perp - q_\beta e_{\perp\mu}^{(\lambda)}g_{\alpha\nu}^\perp - q_\alpha e_{\perp\nu}^{(\lambda)}g_{\beta\mu}^\perp + q_\beta e_{\perp\nu}^{(\lambda)}g_{\alpha\mu}^\perp]T_1(v, qz) \\ &+ e_q\langle\bar{q}q\rangle[q_\mu e_{\perp\alpha}^{(\lambda)}g_{\beta\nu}^\perp - q_\mu e_{\perp\beta}^{(\lambda)}g_{\alpha\nu}^\perp - q_\nu e_{\perp\alpha}^{(\lambda)}g_{\beta\mu}^\perp + q_\nu e_{\perp\beta}^{(\lambda)}g_{\alpha\mu}^\perp]T_2(v, qz) \\ &+ \frac{e_q\langle\bar{q}q\rangle}{qz}[q_\alpha q_\mu e_{\perp\beta}^{(\lambda)}z_\nu - q_\beta q_\mu e_{\perp\alpha}^{(\lambda)}z_\nu - q_\alpha q_\nu e_{\perp\beta}^{(\lambda)}z_\mu + q_\beta q_\nu e_{\perp\alpha}^{(\lambda)}z_\mu]T_3(v, qz) \\ &+ \frac{e_q\langle\bar{q}q\rangle}{qz}[q_\alpha q_\mu e_{\perp\nu}^{(\lambda)}z_\beta - q_\beta q_\mu e_{\perp\nu}^{(\lambda)}z_\alpha - q_\alpha q_\nu e_{\perp\mu}^{(\lambda)}z_\beta + q_\beta q_\nu e_{\perp\mu}^{(\lambda)}z_\alpha]T_4(v, qz),\end{aligned}\quad (4.24)$$

where

$$T_i(v, qz) = \int \mathcal{D}\underline{\alpha} e^{-i(qz)\alpha_v} T_i(\underline{\alpha}). \quad (4.25)$$

The functions $T_i(\underline{\alpha})$ are related to the distributions for a generic EM BG field by integration by parts:

$$\bar{T}_i(\underline{\alpha}) = -2 \int_0^{\alpha_{\bar{q}}} du T_i(u, \alpha_{\bar{q}} + \alpha_q - u). \quad (4.26)$$

Here we imply $T_i(\underline{\alpha}) \equiv T_i(\alpha_{\bar{q}}, \alpha_q)$. The corresponding equivalent definitions of S_γ and T_4^γ are analogous to (4.23) and (4.24), respectively; the relation between \bar{T}_4^γ and T_4^γ is the same as in (4.26).

4.3.2 Conformal expansion of three-particle twist-4 distribution amplitudes

In practical calculations one has to use models for the higher-twist DAs with a few non-perturbative parameters. The major difficulty in constructing such models is that the QCD EOM imply the existence of relations between different amplitudes and that these relations have to be obeyed identically. The standard approach [11, 34] is to expand the amplitudes in contributions of operators with increasing conformal spin and retain only a few leading terms. The reason why this helps is that the QCD EOM are essentially tree-level relations that respect the same symmetries as the QCD Lagrangian. Relations between DAs imposed by EOM are therefore “horizontal” in the sense that they relate contributions of only the same conformal spin. This implies that the EOM can be solved order by order in the conformal expansion and, most importantly, the truncation of the conformal expansion at a certain order is not in conflict with the QCD EOM. In addition,

as familiar from leading-twist, the conformal expansion is consistent with renormalization at LO level: contributions with different conformal spin do not mix with each other to leading-logarithmic accuracy. This ensures that higher-spin contributions omitted in the model at a certain scale will not reappear at different scales through the evolution.

Technically speaking, the conformal expansion of distribution amplitudes is the expansion in orthogonal polynomials that correspond to irreducible representations of the conformal group. A suitable basis is defined in App. A. We have already seen examples of conformal expansion in Eqs. (3.4) and (4.9). The expansion of the three-particle DAs of the photon of twist-4 is completely analogous to that of the ρ , done in [32]. Without going into details we just quote the results:

$$\begin{aligned}
T_1(\underline{\alpha}) &= -120(3\zeta_2 + \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \\
T_2(\underline{\alpha}) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q) \left[(\kappa - \kappa^+) + (\zeta_1 - \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g) \right], \\
T_3(\underline{\alpha}) &= -120(3\zeta_2 - \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \\
T_4(\underline{\alpha}) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q) \left[(\kappa + \kappa^+) + (\zeta_1 + \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g) \right], \\
S(\underline{\alpha}) &= 30\alpha_g^2 \left\{ (\kappa + \kappa^+)(1 - \alpha_g) + (\zeta_1 + \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) \right. \\
&\quad \left. + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)] \right\}, \\
\tilde{S}(\underline{\alpha}) &= -30\alpha_g^2 \left\{ (\kappa - \kappa^+)(1 - \alpha_g) + (\zeta_1 - \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) \right. \\
&\quad \left. + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)] \right\}.
\end{aligned} \tag{4.27}$$

Here terms in κ and κ^+ correspond to the lowest conformal spin-3 and the contributions of ζ s are next-to-leading with conformal spin-4. For the lowest spin the definitions are:

$$\begin{aligned}
\langle 0 | \bar{q}gG_{\mu\nu}q | 0 \rangle_F &= e_q \langle \bar{q}q \rangle (\kappa^+ + \kappa) F_{\mu\nu}, \\
\langle 0 | \bar{q}g\tilde{G}_{\mu\nu}\gamma_5 q | 0 \rangle_F &= e_q \langle \bar{q}q \rangle (\kappa^+ - \kappa) F_{\mu\nu};
\end{aligned} \tag{4.28}$$

κ and κ^+ renormalize multiplicatively [11]:

$$\begin{aligned}
\kappa^+(\mu^2) &= L^{(\gamma^+ - \gamma_{\bar{q}q})/b} \kappa^+(\mu_0^2), \quad \gamma_+ = 3C_A - \frac{5}{3}C_F, \\
\kappa(\mu^2) &= L^{(\gamma^- - \gamma_{\bar{q}q}/b)} \kappa(\mu_0^2), \quad \gamma_- = 4C_A - 3C_F.
\end{aligned} \tag{4.29}$$

The ζ -parameters can be expressed in terms of reduced matrix elements $\langle\langle Q^{(i)} \rangle\rangle$ of local quark-antiquark-gluon operators with one extra covariant derivative and also receive contributions of quark-antiquark-photon operators. One finds [11]

$$\begin{aligned}
\zeta_1 &= \frac{21}{20} \langle\langle Q^{(1)} \rangle\rangle, \quad \zeta_2 = \frac{7}{22} + \frac{7}{22} \langle\langle Q^{(3)} \rangle\rangle - \frac{21}{220} \langle\langle Q^{(1)} \rangle\rangle, \\
\zeta_1^+ &= 7 \langle\langle Q^{(5)} \rangle\rangle, \quad \zeta_2^+ = \frac{7}{4} \langle\langle Q^{(3)} \rangle\rangle,
\end{aligned} \tag{4.30}$$

κ	κ^+	ζ_1	ζ_2	ζ_1^+	ζ_2^+
0.2	0	0.4	0.3	0	0

Table 3: Numerical values of parameters of twist-4 chiral-odd DAs taken from [11]. Renormalization scale is $\mu = 1$ GeV.

where $Q^{(1)}$, $Q^{(3)}$, and $Q^{(5)}$ are multiplicatively renormalizable operators (in pure QCD, that is not including mixing with photon operators) listed in Sec. 4 of Ref. [11]:

$$\begin{aligned} \langle\langle Q^{(i)} \rangle\rangle(\mu^2) &= L^{(\gamma_{Q^{(i)}} - \gamma_{\bar{q}q})/b} \langle\langle Q^{(i)} \rangle\rangle(\mu_0^2) \\ \text{with } \gamma_{Q^{(1)}} &= \frac{11}{2}C_A - 3C_F, \quad \gamma_{Q^{(3)}} = \frac{13}{3}C_F, \quad \gamma_{Q^{(5)}} = 5C_A - \frac{8}{3}C_F. \end{aligned} \quad (4.31)$$

The exact definition of these operators is not important here. Numerical estimates [11] are collected in Tab. 3.

Note that four parameters are defined by three independent matrix elements so that there is one (exact) relation:

$$\zeta_1 + 11\zeta_2 - 2\zeta_2^+ = \frac{7}{2}, \quad (4.32)$$

which is a consequence of the theorem by Ferrara-Grillo-Parisi-Gatto [35], which states that the divergence of a conformal operator vanishes (in a conformally-invariant theory). In the present case, the theorem implies that the divergence of the leading-twist-2 conformal operator with two covariant derivatives vanishes in free theory and, therefore, in full QCD can be expressed in terms of operators including either the gluon or the photon field strength. One obtains [11, 32]

$$\partial_\mu (O_{\xi, \eta\alpha\mu}^2 - O_{\eta, \xi\alpha\mu}^2) = 20(Q_{\alpha, \xi\mu}^{(4)} + Q_{\alpha, \xi\mu}^{(4F)}), \quad (4.33)$$

where $O_{\perp, +++}^2 = \frac{15}{2}\bar{q}\sigma_{\perp+} \overleftrightarrow{D}_+ \overleftrightarrow{D}_+ q - \frac{3}{2}\partial_+^2 \bar{q}\sigma_{\perp+} q$ and the explicit expression for the quark-antiquark-gluon operator $Q_{\alpha, \xi\mu}^{(4)}$ is given in Eq. (4.23) in [11]. $Q_{\alpha, \xi\mu}^{(4F)}$ is obtained from $Q_{\alpha, \xi\mu}^{(4)}$ by the substitution $gG_{\mu\nu} \rightarrow e_q F_{\mu\nu}$. The vacuum expectation value of the l.h.s. of (4.33) in the EM field vanishes, whereas $\langle\langle Q_{\alpha, \xi\mu}^{(4F)} \rangle\rangle$ is equal to a constant times the quark condensate [11]. It follows that the reduced matrix element of the complicated quark-antiquark gluon operator $Q_{\alpha, \xi\mu}^{(4)}$ in the BG field can be calculated exactly in terms of the condensate, and this is how Eq. (4.32) emerges. This relation is of principal importance since it tells that contributions of gluon and photon operators cannot be separated in a meaningful way. Hence the use of an approximation for gluon contributions necessarily entails the use of the corresponding approximation for photon contributions as well. In particular, the quark-antiquark-photon DAs have to be taken into account to the *same* accuracy in the conformal expansion as the quark-antiquark-gluon ones, notwithstanding that the former are known exactly (as given in (4.20)).

The conformal expansion of quark-antiquark-photon DAs is constructed following the approach of Ref. [34]. To this end one has to separate different quark-spin projections.

We define the auxiliary amplitudes

$$\begin{aligned}\langle 0|\bar{q}(z)\gamma_+\gamma_-F_{\mu\nu}(vz)q(-z)|0\rangle &= \langle\bar{q}q\rangle F_{\mu\nu}S^{\uparrow\downarrow}(v,pz), \\ \langle 0|\bar{q}(z)\gamma_-\gamma_+F_{\mu\nu}(vz)q(-z)|0\rangle &= \langle\bar{q}q\rangle F_{\mu\nu}S^{\downarrow\uparrow}(v,pz).\end{aligned}\quad (4.34)$$

Then

$$S_\gamma(\underline{\alpha}) = \frac{1}{2}(S^{\uparrow\downarrow}(\underline{\alpha}) + S^{\downarrow\uparrow}(\underline{\alpha})), \quad T_4^\gamma(\underline{\alpha}) = \frac{1}{2}(S^{\uparrow\downarrow}(\underline{\alpha}) - S^{\downarrow\uparrow}(\underline{\alpha})) = 0. \quad (4.35)$$

The auxiliary amplitudes are expanded in conformal polynomials as

$$\begin{aligned}S^{\uparrow\downarrow}(\underline{\alpha}) &= 60\alpha_{\bar{q}}\alpha_g^2 \left[f_{00} + f_{10} \left(\alpha_g - \frac{3}{2}\alpha_{\bar{q}} \right) + f_{01}(\alpha_g - 3\alpha_{\bar{q}}) \right], \\ S^{\downarrow\uparrow}(\underline{\alpha}) &= 60\alpha_q\alpha_g^2 \left[f_{00} + f_{10} \left(\alpha_g - \frac{3}{2}\alpha_q \right) + f_{01}(\alpha_g - 3\alpha_q) \right].\end{aligned}\quad (4.36)$$

As for the quark-antiquark-gluon DAs, the contributions in f_{00} have conformal spin-3, those in f_{01} and f_{10} conformal spin-4. The constants f_{ij} can be easily obtained by imposing the relations

$$\int \mathcal{D}\underline{\alpha} S^{\uparrow\downarrow}(\underline{\alpha}) = f_{00}, \quad \int \mathcal{D}\underline{\alpha} S^{\uparrow\downarrow}(\underline{\alpha}) \alpha_{q,\bar{q}} = 0, \quad (4.37)$$

and using the exact expressions (4.20) for S_γ and T_4^γ . We find

$$f_{00} = 1, \quad f_{10} = \frac{14}{3}, \quad f_{01} = \frac{7}{3}. \quad (4.38)$$

With (4.35) we then arrive at the following expressions:

$$\begin{aligned}S_\gamma(\underline{\alpha}) &= 30\alpha_g^2 \left[(1 - \alpha_g) + \frac{14}{3} \left\{ \alpha_g(1 - \alpha_g) - \frac{3}{2}(\alpha_{\bar{q}}^2 + \alpha_q^2) \right\} + \frac{7}{3} \{ \alpha_g(1 - \alpha_g) - 6\alpha_{\bar{q}}\alpha_q \} \right] \\ &= 60\alpha_g^2(\alpha_q + \alpha_{\bar{q}})(4 - 7(\alpha_q + \alpha_{\bar{q}})), \\ T_4^\gamma(\underline{\alpha}) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q) \left[1 + \frac{7}{3}(5\alpha_g - 3) + \frac{7}{3}\alpha_g \right] \\ &= 60\alpha_g^2(\alpha_q - \alpha_{\bar{q}})(4 - 7(\alpha_q + \alpha_{\bar{q}})).\end{aligned}\quad (4.39)$$

We would like to stress that the conformal expansion of $T_4^\gamma \equiv 0$ is nontrivial: to any finite order in the expansion one obtains a nonzero result. This complication arises because T_4^γ , in contrast to the chiral-even twist-3 DAs, does not have simple transformation properties under the conformal transformation and corresponds to the difference of two amplitudes $S^{\uparrow\downarrow}$ and $S^{\downarrow\uparrow}$ with fixed spin projections. The symmetry $S^{\uparrow\downarrow} = S^{\downarrow\uparrow}$ is broken to each finite order in the expansion and is only restored in the infinite sum. Note that if N conformal partial waves are taken into account, then the first N moments of T_4^γ vanish. In the next section we will consider an example to illustrate why using the truncated expressions in (4.39) is mandatory for selfconsistency of the OPE. This complication was overlooked in [11, 13] so that the corresponding expressions have to be revised.

4.3.3 Equations of motion and two-particle twist-4 distribution amplitudes

With the three-particle DAs in hand, we are in a position to calculate the two-particle DAs. To this end, it is convenient to use operator identities for the derivatives of the relevant chiral-odd nonlocal operators and take the vacuum averages in the BG field. For example, one obtains¹⁰

$$\begin{aligned}
\partial_\mu \bar{q}(x) \sigma_{\mu\nu} q(-x) &= -i \frac{\partial}{\partial x_\nu} \bar{q}(x) q(-x) + \int_{-1}^1 dv v \bar{q}(x) (gG_{x\nu} + e_q F_{x\nu})(vx) q(-x) \\
&\quad - i \int_{-1}^1 dv \bar{q}(x) x_\rho (gG_{\rho\mu} + e_q F_{\rho\mu})(vx) \sigma_{\mu\nu} q(-x), \\
\frac{\partial}{\partial x_\nu} \bar{q}(x) \sigma_{\mu\nu} q(-x) &= -i \partial_\nu \bar{q}(x) q(-x) + \int_{-1}^1 dv \bar{q}(x) (gG_{x\nu} + e_q F_{x\nu})(vx) q(-x) \\
&\quad - i \int_{-1}^1 dv v \bar{q}(x) (gG_{x\mu} + e_q F_{x\mu})(vx) \sigma_{\mu\nu} q(-x).
\end{aligned} \tag{4.40}$$

These identities can be obtained from the ones derived in [31, 32] by the replacement $gG \rightarrow gG + e_q F$. Taking matrix-elements and comparing with the definitions in (2.19) and (4.21), one obtains the following relations

$$\begin{aligned}
h_\gamma(u) &= -\frac{d}{du} \int_0^u d\alpha_q \int_0^{\bar{u}} d\alpha_{\bar{q}} \frac{2}{\alpha_g} \left[\frac{\alpha_q - \alpha_{\bar{q}} - (2u-1)}{\alpha_g} [S(\underline{\alpha}) + S_\gamma(\underline{\alpha})] + T_3(\underline{\alpha}) - T_2(\underline{\alpha}) \right], \\
\mathbb{A}(u) &= 2 \int_0^u dv (4u+1-6v) h_\gamma(v) \\
&\quad - 4 \int_0^u d\alpha_q \int_0^{\bar{u}} d\alpha_{\bar{q}} \frac{1}{\alpha_g} \left[\frac{\alpha_q - \alpha_{\bar{q}} - (2u-1)}{\alpha_g} (T_2(\underline{\alpha}) - T_3(\underline{\alpha})) - [S(\underline{\alpha}) + S_\gamma(\underline{\alpha})] \right],
\end{aligned} \tag{4.41}$$

from the first and from the second of the identities in (4.40), respectively. These relations, again, correspond to (A.13) and (A.14) in [31] with the replacement $S \rightarrow S + S_\gamma$. Inserting expressions for the three-particle DAs at next-to-leading order in the conformal expansion and using (4.32), we obtain for h_γ :

$$h_\gamma(u) = -10(1 + 2\kappa^+) C_2^{1/2} (2u-1). \tag{4.42}$$

The cancellation of contributions $\sim \zeta_i$ is a consequence of conformal symmetry. To see this, notice that h_γ corresponds to the configuration with both the quark and the antiquark having “minus” light-cone projections and therefore a well-defined conformal expansion in terms of Gegenbauer-polynomials $C_{j-1}^{1/2}$ where j is the conformal spin, cf. [34, 31]. The symmetry $h_\gamma(u) = h_\gamma(1-u)$ implies that only odd conformal spins can contribute, and in addition the contribution of the lowest possible spin $j=1$ vanishes by EOM. Hence $j=3, 5, \dots$, and in particular the result in (4.42) corresponds to $j=3$. The ζ -contributions

¹⁰Here and below we use ∂_μ to denote the derivative of a nonlocal operator with respect to the total translation, cf. [15].

to three-particle DAs have $j = 4$; they cannot appear in the expansion of h_γ and indeed cancel by virtue of (4.32).

The situation with \mathbb{A} is different, since this function is defined off the light-cone and its conformal expansion is more complicated. Using the result in (4.42) and the second relation in (4.41) we obtain

$$\begin{aligned} \mathbb{A}(u) = & 40u^2\bar{u}^2 \{3\kappa - \kappa^+ + 1\} + 8(\zeta_2^+ - 3\zeta_2) \times \\ & \times (u\bar{u}(2 + 13u\bar{u}) + 2u^3(10 - 15u + 6u^2) \ln u + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2) \ln \bar{u}). \end{aligned} \quad (4.43)$$

We emphasize that our results for the DAs h_γ and \mathbb{A} are consistent with the approximations adopted for the three-particle DAs; that is, this set of photon distribution amplitudes satisfies all exact QCD identities that are consequence of EOM. This issue is nontrivial. For example, consider an important operator identity which is an extension of Eq. (24) in [13]:¹¹

$$\begin{aligned} \bar{q}(x)\sigma_{\mu\nu}q(-x) = & \int_0^1 duu \left\{ \frac{\partial}{\partial x_\nu} \bar{q}(ux)\sigma_{\mu x}q(-ux) - (\mu \leftrightarrow \nu) \right\} - \varepsilon_{\mu\nu x\xi} \partial_\xi \int_0^1 duu^2 \bar{q}(ux)\gamma_5 q(-ux) \\ & - i\varepsilon_{\mu\nu x\xi} \int_0^1 duu^2 \int_{-u}^u dv \bar{q}(ux) \{gG_{x\xi}(vx) + e_q F_{x\xi}(vx)\} \gamma_5 q(-ux) \\ & + i \int_0^1 duu \int_{-u}^u dv v \bar{q}(ux) \{[gG_{\mu x}(vx) + e_q F_{\mu x}(vx)]\sigma^{\nu x} - (\mu \leftrightarrow \nu)\} q(-ux). \end{aligned} \quad (4.44)$$

Taking the vacuum average in the BG field and using the definitions of photon DAs, this implies one more relation

$$\begin{aligned} \mathbb{A}(u) = & 2 \int_0^u dv (2v - 1) h_\gamma(v) \\ & - 4 \int_0^u d\alpha_q \int_0^{\bar{u}} d\alpha_{\bar{q}} \frac{1}{\alpha_g} \left[\frac{\alpha_q - \alpha_{\bar{q}} - (2u - 1)}{\alpha_g} (T_4(\underline{\alpha}) + T_4^\gamma(\underline{\alpha}) - T_3(\underline{\alpha})) + \tilde{S}(\underline{\alpha}) \right]. \end{aligned} \quad (4.45)$$

One can check that the resulting expression for \mathbb{A} coincides with the result given in (4.43) if and only if full expressions for the three-particle DAs and in particular the nonzero result for T_4^γ (4.39) are used. In Ref. [13] the DAs h_γ and \mathbb{A} were derived using a truncated version of (4.44) neglecting T_4^γ and quark-antiquark-gluon operators. In view of relation (4.32) this approximation is inconsistent and the resulting DAs are superseded by the ones obtained in this paper.

5 Summary and Conclusions

In this paper we have given the first comprehensive analysis of photon distribution amplitudes. The special interest (and complication) springs from the interplay between hard

¹¹This identity is very useful in practical calculations since it allows to express the chiral-odd operator with two free Lorentz-indices through a simpler nonlocal operator with one open index and three-particle operators of higher twist.

(perturbative) and soft (nonperturbative) contributions to the photon structure, as discussed in Sec. 2.2. This interplay becomes nontrivial in higher twists, as QCD equations of motion receive additional contributions from photon operators. (Alternatively, one may leave equations of motion intact, but take into account contact terms.)

We have defined and classified all two- and three-particle distribution amplitudes of the photon up to twist-4. For the chiral-odd ones, we could largely draw on the results obtained in Ref. [11], the remaining definitions resemble those for the ρ -meson discussed in [31, 32]. The main theoretical result of our analysis is the conformal expansion of DAs in presence of the background field. We provide a selfconsistent approximation for photon DAs up to twist-4 that contains a minimal number of nonperturbative parameters and is consistent with the exact QCD equations of motion. The final expressions are collected in Eqs. (4.9) (twist-3) and (4.27), (4.39), (4.42), (4.43) (twist-4). The numerical values of the parameters are estimated, when possible, using QCD sum rules. In particular, we give a new estimate of the magnetic susceptibility of the quark condensate and derive a sum rule for the normalization of twist-three distributions. All parameters are accessible to lattice-simulations, and we hope that the lattice-community will take up the task and provide more accurate estimates.

We believe that the results of this work can be applied to the studies of hard exclusive processes involving photon emission (or absorption) in the framework of QCD factorization. A few examples are mentioned in the introduction. A more detailed discussion and concrete calculations go beyond the tasks of this paper.

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Appendices

A The Conformal Basis

For a generic three-particle operator a “conformal basis” can be constructed as follows [36, 37]: conformal symmetry allows one to fix the total three-particle conformal spin $J = j_1 + j_2 + j_3 + N$ of a state. We define a set of functions $Y_{Jj}^{(12)3}$ by requiring that, in addition to a fixed J , they also have a definite value of the conformal spin in the quark-antiquark channel (12), for definiteness: $j = j_1 + j_2 + n$ with $n = 0, \dots, N$.

Taken together, these two conditions determine the polynomials $Y_{Jj}^{(12)3}$ uniquely and yield the following expression:

$$Y_{Jj}^{(12)3}(\alpha_i) = (1 - \alpha_3)^{j-j_1-j_2} P_{J-j-j_3}^{(2j_3-1, 2j-1)}(1 - 2\alpha_3) P_{j-j_1-j_2}^{(2j_1-1, 2j_2-1)} \left(\frac{\alpha_2 - \alpha_1}{1 - \alpha_3} \right). \quad (\text{A.1})$$

Here $P_n^{(\alpha, \beta)}(x)$ is the Jacobi-polynomial.

The basis functions $Y_{Jj}^{(12)3}(\alpha_i)$ are mutually orthogonal with respect to the conformal scalar product:

$$\int_0^1 \mathcal{D}\underline{\alpha} \alpha_1^{2j_1-1} \alpha_2^{2j_2-1} \alpha_3^{2j_3-1} Y_{Jj}^{(12)3}(\alpha_i) Y_{Jj'}^{(12)3}(\alpha_i) = \mathcal{N}_{Jj} \delta_{JJ'} \delta_{jj'}, \quad (\text{A.2})$$

where

$$\mathcal{N}_{Jj} = \frac{\Gamma(j+j_1-j_2)\Gamma(j-j_1+j_2)}{\Gamma(j-j_1-j_2+1)\Gamma(j+j_1+j_2-1)(2j-1)} \frac{\Gamma(J-j+j_3)\Gamma(J+j-j_3)}{\Gamma(J-j-j_3+1)\Gamma(J+j+j_3-1)(2J-1)}. \quad (\text{A.3})$$

The above construction of the conformal basis involves the obvious ambiguity in what order to couple the spins of partons to the total spin J . Choosing a different two-particle channel one obtains a different conformal basis related to the original one through the matrix Ω of Racah $6j$ -symbols of $\text{SL}(2, \mathbb{R})$, e.g.

$$Y_{Jj}^{(31)2}(x_i) = \sum_{j_1+j_2 \leq j' \leq J-j_3} \Omega_{jj'}(J) Y_{Jj'}^{(12)3}(x_i). \quad (\text{A.4})$$

The properties of the Racah $6j$ -symbols as well as explicit expressions in terms of the generalized hypergeometric series ${}_4F_3(1)$ are summarized in [37].

B QCD Sum Rules

Given the considerable time that has passed since the existing numerical estimates of the parameters of photon DAs were obtained [27, 28, 11], we feel that the corresponding QCD sum rules deserve a fresh look. Some new results are also included: we calculate the radiative correction to the leading-twist sum rule and derive a sum rule for the normalization of twist-3 distributions.

B.1 The leading twist-2 distribution amplitude

The QCD sum rules for the leading-twist photon DA $\phi_\gamma(u)$ are derived from the correlation function of the nonlocal tensor operator on the light-cone with the vector current. For massless quarks:

$$i \int d^4y e^{-iqy} \langle 0 | T[\bar{q}\gamma_z q](y) \bar{q}(0) \sigma_{\mu z} [0, z] q(z) | 0 \rangle = i z_\mu \frac{qz}{q^2} \langle \bar{q}q \rangle \int_0^1 du e^{-iquz} \Phi(u, q^2) \quad (\text{B.1})$$

with

$$\Phi(u, q^2) = -[\delta(u) + \delta(1-u)] - \frac{\alpha_s C_F}{2\pi} F(u) - \frac{1}{3} \frac{m_0^2}{q^2} [\delta(u) + \delta(1-u) + \delta'(u) + \delta'(1-u)] + \mathcal{O}(1/q^4) \quad (\text{B.2})$$

where $\delta'(x) = (d/dx)\delta(x)$ and

$$F(u) = \left(\ln \frac{-q^2}{\mu^2} - 1 \right) \left\{ 2\delta(u) + 2\delta(\bar{u}) - \left[\frac{\bar{u}}{u} \right]_+ - \left[\frac{u}{\bar{u}} \right]_+ \right\} - \left[\frac{\bar{u} \ln(u\bar{u})}{u} \right]_+ - \left[\frac{u \ln(u\bar{u})}{\bar{u}} \right]_+. \quad (\text{B.3})$$

Here the $[]_+$ prescription is defined as usual

$$[f(u)]_+ = f(u) - \delta(u_0) \int_0^1 dw f(w),$$

where $u_0 = 0$ or $u_0 = 1$ is the position of the singularity, and m_0^2 parametrizes the mixed condensate: $\langle \bar{q}\sigma g G q \rangle \equiv m_0^2 \langle \bar{q}q \rangle$. The radiative correction to the quark condensate in (B.3) is a new result¹². By expanding (B.1) around $z = 0$ in terms of conformal invariant local operators one obtains the functions $A_n(q^2)$ given in Eq. (4.7) in [11], up to the terms in α_s .

B.1.1 Magnetic susceptibility

The magnetic susceptibility of the quark condensate, χ , is obtained from the correlation function

$$q^2 \chi(q^2) = \int_0^1 du \Phi(u, q^2) = -2 + \frac{8}{3} \frac{\alpha_s}{\pi} \left(\ln \frac{\mu^2}{-q^2} + 1 \right) - \frac{2}{3} \frac{m_0^2}{q^2} + 0 \cdot \frac{\langle g^2 G^2 \rangle}{q^4} + \dots, \quad (\text{B.4})$$

where we have also indicated that the gluon condensate contribution vanishes in factorization approximation [28]. We write the physical spectral density of this correlation function as a sum over several narrow resonances plus a smooth continuum starting at a threshold s_0 . Assuming quark-hadron duality, the continuum contribution can be represented by the perturbative imaginary part of the radiative correction, so that

$$\chi(q^2) \langle \bar{q}q \rangle = - \sum_i \frac{m_i f_i f_i^T}{m_i^2 - q^2} + \frac{8\alpha_s}{3\pi} \langle \bar{q}q \rangle \int_{s_0}^{\infty} ds \frac{1}{s(s - q^2)}, \quad (\text{B.5})$$

where f_i and f_i^T denote the couplings of the resonance i to the vector and tensor current, and are defined as in (3.7). The factor $8/3$ in front of the integral is related to the anomalous dimensions defined in (3.4): $1/2(\gamma_0 - \gamma_{\langle \bar{q}q \rangle}) = 8/3$. Once these parameters are fixed, the magnetic susceptibility is obtained from (B.5) as $\chi = \chi(0)$.

In a first approximation one can retain the contribution of the lowest-lying ρ -meson state only and use the same value of the continuum threshold as obtained in the sum rules for the correlation function of vector currents, $s_0 = 1.5 \text{ GeV}^2$ [30]. Using the QCD sum rule estimate $f_\rho^\perp = 160 \text{ MeV}$ one obtains

$$\chi(1 \text{ GeV}) = - \frac{f_\rho f_\rho^\perp}{m_\rho \langle \bar{q}q \rangle} + \frac{8\alpha_s}{3\pi} \frac{1}{s_0} \simeq 3.0 \text{ GeV}^{-2}, \quad (\text{B.6})$$

¹²The expression given in [23] for the local limit $z \rightarrow 0$ contains a typo.

which is an improvement over the pure VDM estimate in (3.9). Including more resonances and at the same time increasing the continuum threshold, one may hope to get a better accuracy. The corresponding sum rules can be constructed in two different ways: either matching the expansion of (B.5) for $q^2 \rightarrow -\infty$ to the power series in (B.4) [28] or performing the Borel-transformation of both expressions and matching the results for intermediate values of the Borel-parameter of the order of a few GeV [27]. Once the parameters are fixed, the magnetic susceptibility is obtained from (B.5) as $\chi = \chi(0)$.

Following the first procedure we obtain the so-called local-duality sum rules:

$$\begin{aligned}\sum m_i f_i f_i^T(\mu) &= -2\langle\bar{q}q\rangle(\mu) \left[1 + \frac{4\alpha_s(\mu)}{3\pi} \left(\ln \frac{s_0}{\mu^2} - 1 \right) \right], \\ \sum m_i^3 f_i f_i^T(\mu) &= -2\langle\bar{q}q\rangle(\mu) \left[\frac{4\alpha_s(\mu)}{3\pi} s_0 + \frac{m_0^2(\mu)}{3} \right], \\ \sum m_i^5 f_i f_i^T(\mu) &= -4\langle\bar{q}q\rangle(\mu) \left[\frac{\alpha_s(\mu)}{3\pi} s_0^2 + 0 \cdot \langle G^2 \rangle \right].\end{aligned}\tag{B.7}$$

For the couplings of the ground-state resonance $\rho(770)$ one can use the values quoted in (3.7). The three local-duality sum rules in (B.7) can be used to determine the contributions of three more resonances, so that we truncate the sums in (B.7) after four terms. Experimentally, the three lowest-lying resonances $\rho(770)$, $\rho(1450)$ and $\rho(1700)$ are established [26]. The magnetic susceptibility $\chi(\mu)$ then exhibits a mild dependence on s_0 and on the unknown mass of the fourth resonance m_4 . As an example, we show in Fig. 3 $\chi(1 \text{ GeV})$ as function of m_4 with s_0 fixed as $s_0 = (m_4 + 0.1 \text{ GeV})^2$ (solid line) and $(m_4 + 0.2 \text{ GeV})^2$ (dashed line). In this calculation we use $\langle\bar{q}q\rangle(1 \text{ GeV}) = (-0.25 \text{ GeV})^3$, $m_0^2(1 \text{ GeV}) = 0.65 \text{ GeV}^2$, cf. [23], and the value $\alpha_s(1 \text{ GeV}) = 0.51$.

Neglecting for the moment the uncertainty of $f_1 f_1^T$ and the local-duality approximation itself, we obtain from local-duality sum rules with four resonances

$$\chi(1 \text{ GeV}) = (3.08 \pm 0.02) \text{ GeV}^{-2}.\tag{B.8}$$

For a typical value of m_4 , the breakdown of the total number in contributions from the different resonances and the continuum is $2.73 + 0.30 - 0.07 + 0.04 + 0.10$, which shows a clear dominance of $\rho(770)$. Neglecting radiative corrections altogether, we would find $\chi = (3.3 \pm 0.1) \text{ GeV}^{-2}$.

As a consistency check, we relax the condition that the known value of $f_1 f_1^T$ be used as input and include only three resonances in the sums in (B.7). We then find $f_1^T(1 \text{ GeV}) = (170 \pm 7) \text{ MeV}$ for $m_3^2 \leq s_0 \leq 5 \text{ GeV}^2$, which is consistent with the result from Borel sum-rules quoted above, and $\chi(1 \text{ GeV}) = (3.20 \pm 0.15) \text{ GeV}^{-2}$, in complete agreement with (B.8).

Alternatively, and this constitutes the 2nd procedure, one investigates the Borel sum-rule

$$\begin{aligned}m_\rho f_\rho f_\rho^T e^{-m_\rho^2/M^2} + m_{\rho'} f_{\rho'} f_{\rho'}^T e^{-m_{\rho'}^2/M^2} = \\ = -2\langle\bar{q}q\rangle \left[1 + \frac{4}{3} \frac{\alpha_s}{\pi} \left(\ln \frac{M^2}{\mu^2} - \gamma_E - 1 - \int_{s_0}^{\infty} \frac{ds}{s} e^{-s/M^2} \right) - \frac{1}{3} \frac{m_0^2}{M^2} + 0 \cdot \frac{\langle G^2 \rangle}{M^4} \right],\end{aligned}\tag{B.9}$$

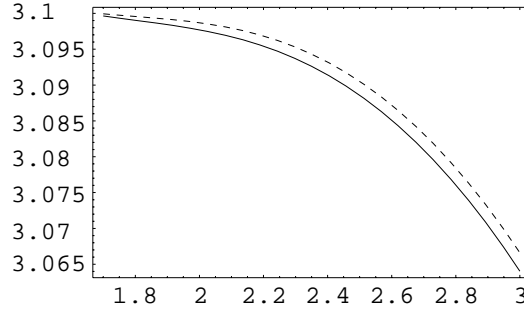


Figure 3: $\chi(1 \text{ GeV})$ as function of m_4 with s_0 fixed as $s_0 = (m_4 + 0.1 \text{ GeV})^2$ (solid line) and $(m_4 + 0.2 \text{ GeV})^2$ (dashed line).

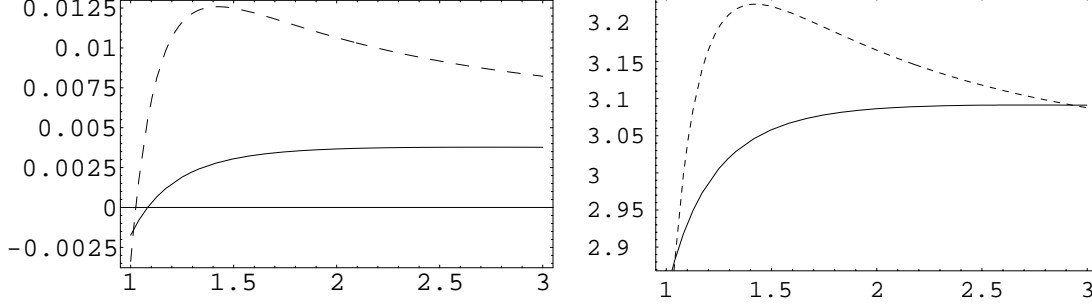


Figure 4: (a) Coupling $f_{\rho'} f_{\rho'}^T$ from the Borel sum-rule (B.9) as a function of the Borel-parameter. Solid line: $m_{\rho'} = 1.4 \text{ GeV}$, dashed line: $m_{\rho'} = 2 \text{ GeV}$. $s_0 = (m_{\rho'} + 0.1 \text{ GeV})^2$. (b) $\chi(1 \text{ GeV})$ from (B.5) with $q^2 = 0$ with two resonances, $\rho(770)$ and an effective ρ' , as a function of the Borel-parameter. Identification of curves as in (a).

where for simplicity we have substituted the sum over higher resonances by the contribution of one effective state ρ' and consider the mass $m_{\rho'}$ as a free parameter. The sum rule (B.9) can be used to extract the product $f_{\rho'} f_{\rho'}^T$ as a function of M^2 , s_0 and $m_{\rho'}$. We find that for $m_{\rho'} \geq 1.4 \text{ GeV}$ there is a reasonable stability-plateau in the Borel-parameter M^2 , as shown in Fig. 4. Varying $m_{\rho'}$ between 1.4 and 2.0 GeV and fixing the continuum threshold as $s_0 = (m_{\rho'} + 0.1 \text{ GeV})^2$, we find from (B.5)

$$\chi(1 \text{ GeV}) = (3.15 \pm 0.10) \text{ GeV}^{-2}, \quad (\text{B.10})$$

which is perfectly consistent with the value obtained from local-duality sum-rules.

We are now in a position to quote our final value for $\chi(1 \text{ GeV})$. Averaging over the results from local-duality sum rules with three and four resonances, and the result from Borel sum-rules, we obtain the central value 3.15 GeV^{-2} . In estimating the uncertainty, we take into account that the $\rho(770)$ is the dominant contribution, so that $\Delta\chi/\chi = \Delta(f_{\rho} f_{\rho}^T)/(f_{\rho} f_{\rho}^T)$ in good approximation. Adding in quadrature the uncertainties from $\Delta f_{\rho} = 10 \text{ MeV}$, $\Delta f_{\rho}^T = 10 \text{ MeV}$ and the uncertainty quoted in (B.10), we finally arrive at the estimate quoted in (3.13).

B.1.2 Higher moments

Let us now turn to the shape of the photon DA. At first sight, assuming the vector-dominance approximation in (B.1) $1/(-q^2) \rightarrow 1/m_\rho^2$ at $q^2 \rightarrow 0$, one is led to the conclusion that the photon DA should be strongly peaked at the end-points, with the quark condensate contribution $\delta(u) + \delta(1-u)$ somewhat smoothened by contributions of condensates of higher dimension. We start by giving a qualitative argument why this picture is wrong.

To this end, it is convenient to consider the Gegenbauer-moments of the correlation function (B.1) defined as

$$\begin{aligned}\chi_n(q^2) &= \frac{1}{q^2} \frac{2(2n+3)}{3(n+1)(n+2)} \int_0^1 du C_n^{3/2}(2u-1) \Phi(u, q^2) \\ &= -\frac{2(2n+3)}{3} \left[\frac{1}{q^2} + \frac{(n+1)(n+2)}{6} \frac{m_0^2}{q^4} + \mathcal{O}(n^4/q^6) \right],\end{aligned}\quad (\text{B.11})$$

where for simplicity we suppress radiative corrections. Obviously $\chi_0(q^2) = \chi(q^2)$, and the coefficients ϕ_n in the expansion of the photon DA over the Gegenbauer-polynomials (3.4) are given by $\phi_n = \chi_n(q^2 = 0)/\chi$. Assuming that the correlation function χ_n for arbitrary n is saturated by the contribution of the ρ -meson and using (3.11), one obtains

$$\phi_n^{\text{VDM}} = \frac{(2n+3)}{3} \quad (\text{B.12})$$

and

$$\phi_\gamma(u) = 6u\bar{u} \sum_{n \text{ even}} \frac{1}{3} (2n+3) C_n^{3/2}(2u-1) = \frac{1}{2} [\delta(u) + \delta(\bar{u})], \quad (\text{B.13})$$

as expected.

To see why this is wrong, consider the expansion of the VDM-type contribution in a power series

$$\chi_n(q^2) \simeq \frac{1}{m^2 - q^2} = -\frac{1}{q^2} - \frac{m^2}{q^4} + \dots \quad (\text{B.14})$$

The comparison of the two expansions (B.11) and (B.14) shows that in order for them to be consistent the mass of the resonance has to grow rapidly with n : $m_n \sim n$. Alternatively, one can say that the low-mass ρ -meson contribution becomes irrelevant for large n .

To see what this means, let us generalize the vector-dominance approximation by allowing the effective mass of the resonance to be n -dependent and, in particular, choose the mass such as to reproduce the first two terms in the operator product expansion:

$$m_{n,\text{eff}}^2 = m_0^2(n+1)(n+2)/6. \quad (\text{B.15})$$

It is easy to see that in this approximation

$$\phi_n = \frac{(2n+3)}{3} \frac{m_{0,\text{eff}}^2}{m_{n,\text{eff}}^2} = \frac{2(2n+3)}{3(n+1)(n+2)}. \quad (\text{B.16})$$

One obtains in this model

$$\phi_\gamma(u) = 6u\bar{u} \sum_{n \text{ even}} \frac{2(2n+3)}{3(n+1)(n+2)} C_n^{3/2}(2u-1) = 1, \quad (\text{B.17})$$

that is a flat DA with no momentum fraction dependence: the peaks at the end-points have disappeared. Note that our argumentation only invokes the structure of nonperturbative power-like corrections in the operator product expansion. Taking into account perturbative gluon radiation is expected to further suppress the end-point behaviour, as illustrated in Fig. 2 for the example of the instanton model.

Trying to be somewhat more quantitative, we can write the generalization of (B.5) to arbitrary Gegenbauer-moments:

$$\chi_n(q^2)\langle\bar{q}q\rangle = -\sum_i \frac{m_i f_i f_i^T \phi_{i,n}}{m_i^2 - q^2} + \frac{2}{3} (2n+3) \left(\sum_{j=1}^{n+1} \frac{1}{j} \right) \frac{\alpha_s}{\pi} C_F \langle\bar{q}q\rangle \int_{s_0}^{\infty} ds \frac{1}{s(s-q^2)}, \quad (\text{B.18})$$

where $\phi_{i,n}$ is the n th coefficient in the Gegenbauer-expansion of the DA of the i th resonance. Note that $\gamma_n - \gamma_{\langle\bar{q}q\rangle} = 4C_F \sum_{j=1}^{n+1} (1/j)$. For $n=2$, one can construct local-duality sum rules:

$$\begin{aligned} \sum m_i f_i f_i^T(\mu) \phi_{i,2}(\mu) &= -\frac{14}{3} \langle\bar{q}q\rangle(\mu) \left[1 + \frac{4}{3} \frac{\alpha_s(\mu)}{\pi} \left(\frac{11}{6} \ln \frac{s_0}{\mu^2} - \frac{131}{36} \right) \right], \\ \sum m_i^3 f_i f_i^T(\mu) \phi_{i,2}(\mu) &= -\frac{14}{3} \langle\bar{q}q\rangle(\mu) \left[\frac{22}{9} \frac{\alpha_s(\mu)}{\pi} s_0 + 2m_0^2(\mu) \right], \end{aligned} \quad (\text{B.19})$$

where we suppress a third sum rule, analogous to the 3rd equation in (B.7), because the factorization approximation for the gluon condensate contribution is known to be unreliable.

A comparison with (B.7) shows that the expression on the r.-h.s. of the first sum rule is by a factor 1.36 larger (assuming $s_0 = 2.5 \text{ GeV}^2$), whereas the r.-h.s. of the second sum rule increases by roughly a factor 3. This is consistent with the pattern suggested above, that higher resonances start to play a decisive rôle. Using a model with two resonances, $\rho(770)$ and $\rho(1450)$, and the continuum threshold $s_0 = 2.5 \text{ GeV}^2$ one obtains the small value $\phi_2 = 0.07$. The problem with this procedure is that the fit “wants” a negative value of the second Gegenbauer-coefficient $\phi_{\rho,2}$, which contradicts a more direct QCD sum rule evaluation of this coefficient from diagonal sum-rules [23]. If the value [23], $\phi_{\rho,2} = 0.2 \pm 0.1$, is enforced, and, say, the mass of the resonance is taken as free parameter, the result for ϕ_2 increases by almost an order of magnitude. In our point of view such an instability indicates that no reliable conclusion can be drawn on the value of ϕ_2 in the QCD sum rule approach.

B.2 Twist-3 Distributions

To leading-order accuracy of the conformal expansion we only need the parameter $f_{3\gamma}$ defined in (4.1). It can be estimated from the correlation function

$$i \int d^4 x e^{-iqx} \langle 0 | T \bar{q}(0) \tilde{G}_{z\nu}(0) \gamma_z \gamma_5 q(0) \bar{q}(x) \gamma_\beta q(x) | 0 \rangle = f_{3\gamma}(q^2)(qz)(z_\beta q_\nu - (qz)g_{\nu\beta}) + \dots, \quad (\text{B.20})$$

where the Lorentz-structure is chosen in order to pick up the relevant coupling of twist-3 corresponding to the transverse projection $G_{z\nu} \rightarrow G_{z\perp}$ and to suppress contributions

of twist-4. $f_{3\gamma}$ is obtained as $f_{3\gamma}(0)$. The operator product expansion of $f_{3\gamma}(q^2)$, up to contributions of dimension 6, reads

$$f_{3\gamma}(q^2) = \frac{\alpha_s}{4\pi} \frac{q^2}{\pi^2} \left(\frac{1}{36} \ln \frac{-q^2}{\mu^2} + \text{const.} \right) + \frac{1}{24q^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle - \frac{8}{9} \frac{\pi \alpha_s \langle \bar{q}q \rangle^2}{q^4}, \quad (\text{B.21})$$

and the ρ -meson contribution can be obtained by a standard QCD sum-rule calculation,

$$m_\rho^2 f_\rho^2 \zeta_{3\rho} e^{-m_\rho^2/M^2} = \frac{\alpha_s}{4\pi} \frac{1}{36\pi^2} \int_0^{s_0} ds s e^{-s/M^2} + \frac{1}{24} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + \frac{8}{9} \frac{\pi \alpha_s \langle \bar{q}q \rangle^2}{M^2}. \quad (\text{B.22})$$

Using $s_0 = 1.5 \text{ GeV}^2$ and $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012 \text{ GeV}^4$ [30], we obtain from this sum rule (at the normalization point $\mu = 1 \text{ GeV}$)

$$\zeta_{3\rho}(1 \text{ GeV}) = 0.10 \pm 0.05, \quad (\text{B.23})$$

which is by a factor 3 larger than the result quoted in [31] and obtained in Ref. [33] from the diagonal correlation function of two quark-antiquark-gluon operators.

In vector-dominance approximation $f_{3\gamma} = -f_\rho^2 \zeta_{3\rho}$. Including the continuum contributions, $f_{3\gamma}$ is obtained as¹³

$$f_{3\gamma}(0) = -f_\rho^2 \zeta_{3\rho} + \frac{\alpha_s}{4\pi} \frac{s_0}{\pi^2} \frac{1}{36}. \quad (\text{B.24})$$

Using the value of $\zeta_{3\rho}$ given in (B.23) we obtain the estimate (at the normalization point $\mu = 1 \text{ GeV}$)

$$f_{3\gamma} = -(0.0039 \pm 0.0020) \text{ GeV}^2 \quad (\text{B.25})$$

with $s_0 = 1.5 \text{ GeV}^2$. The correction to the vector-dominance approximation is small, $\sim 5\%$.

To next-to-leading order in the conformal spin, we have two parameters, ω_γ^A and ω_γ^V , which are defined by the following matrix-elements:

$$\begin{aligned} \langle 0 | \bar{q} \not{D} (g G_{\alpha\nu} z^\alpha (i \vec{D} z) - (i \vec{D} z) g G_{\alpha\nu} z^\alpha) q | 0 \rangle_F &= e_q (pz)^2 F_{\nu z} f_{3\gamma} \frac{3}{28} \omega_\gamma^V + O(z_\nu), \\ \langle 0 | \bar{q} \not{D} \gamma_5 \left[i D z, g \tilde{G}_{\mu\nu} z^\mu \right] q - \frac{3}{7} (i \partial z) \bar{q} \not{D} \gamma_5 g \tilde{G}_{\mu\nu} z^\mu q | 0 \rangle_F &= e_q i (pz)^2 F_{\nu z} f_{3\gamma} \frac{3}{28} \omega_\gamma^A + O(z_\nu), \end{aligned} \quad (\text{B.26})$$

where $[\cdot, \cdot]$ stands for the commutator. In the vector-dominance approximation $\omega_\gamma^A = \omega_{3\rho}^A$ and $\omega_\gamma^V = \omega_{3\rho}^V$. The numbers given in (4.10) are quoted from [33] and have to be taken with caution. Since even for leading twist we have been not able to draw reliable conclusions on similar parameters from the sum rules, we consider the given numbers as an order-of-magnitude estimate. From the general experience of similar calculations we expect that this is rather an upper bound.

¹³In order to obtain this formula, we express the continuum contribution to (B.21) via a dispersion relation $\sim \int_{s_0}^{\mu_F^2} ds = \mu_F^2 - s_0$, where μ_F is the factorization scale. The quadratically divergent term $\sim \mu_F^2$ matches the contribution of low momenta in the perturbative contributions of twist-1 in (2.18) which have to be subtracted to avoid double counting. We prefer to define perturbative contributions in such a way as to include all momenta down to zero, and therefore have to omit quadratically divergent parts of the higher-twist operators. Thus we only keep the term in s_0 .

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